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ABSTRACT

It has long been thought that the reversed Cherenkov radiation is impossible in homogeneous media with a positive refractive index n. Here, we break this long-held belief by revealing the possibility of creating reversed Cherenkov radiation from homogeneous positive-index moving media. The underlying mechanism is essentially related to the Fizeau–Fresnel drag effect, which provides a unique route to drag the emitted light in the direction of the moving medium and thus enables the possibility of dragging the emitted light in the opposite direction of the moving charged particle. Moreover, we discover the existence of a threshold for the velocity v_{medium} of moving media, only above which, namely, $v_{\mathrm{medium}} > c/n^2$, the reversed Cherenkov radiation may emerge, where c is the velocity of light in vacuum. Particularly, we find that the reversed Cherenkov radiation inside superluminal moving media (i.e., $v_{\mathrm{medium}} > c/n$) can become thresholdless for the velocity of moving charged particles.

Relativistic motion is widely known to have peculiar effects across all domains of physics,[1–9] especially on the electromagnetic interactions of charged particles. One paradigmatic phenomenon is the Cherenkov radiation,[10–16] which is essentially induced by the relative motion between charged particles and the surrounding medium with a refractive index n. Specifically, Cherenkov radiation emerges when a charged particle moves with a velocity v_e exceeding the so-called Cherenkov threshold, namely, the phase velocity c/n of light in the surrounding stationary medium, where c is the velocity of light in vacuum. One powerful implication of Cherenkov radiation is its directionality across a broad spectral regime, ranging from microwave to terahertz, infrared, visible, ultraviolet, and X-ray.[17–19] Accordingly, Cherenkov radiation is of paramount importance to enormous practical applications,[20–23] including particle detectors for the identification of high-energy particles,[24–29] light sources at previously hard-to-reach frequencies,[30–33] bio-medical imaging,[34–36] and photodynamic therapy.[37–39]

Soon after the experimental discovery of Cherenkov radiation by Cherenkov under the guidance of Vavilov in 1934,[40] their finding was successfully interpreted by Frank and Tamm in 1937 based on the classic electrodynamic theory.[41] According to the well-known Frank–Tamm formula $\cos\theta = c/nv_e$ for stationary media,[42] the Cherenkov radiation angle θ is uniquely related to the particle velocity v_e. Moreover, the conventional Cherenkov radiation generally has $\theta < 90^\circ$ in stationary positive-index media with $n > 0$. Under this scenario, the emitted light would propagate in the same direction as the emitting particle.

In contrast, the reversed Cherenkov radiation[43–46] has $\theta > 90^\circ$, so that the emitted light would propagate in the reversed or opposite direction of particle motion. According to the pioneering prediction of Veselago in 1968,[43] the realization of reversed Cherenkov radiation requires novel materials, namely, materials with a negative refractive index. As a result, both theoretical prediction and experimental observation of reversed Cherenkov radiation are so far limited to the framework of unconventional homogeneous materials (e.g., negative-index materials[47–51] and anisotropic materials[52–56]) or judiciously designed inhomogeneous structures (e.g., gain slabs[57] and photonic crystals[58–60]). In fact, it is widely believed that the reversed Cherenkov radiation is impossible in homogeneous positive-index media. Here, we reveal a universal route to create the reversed Cherenkov radiation from homogeneous positive-index moving media. Our finding thus breaks this long-held tenet.

In general, the surrounding medium is not necessarily stationary but can be in motion. The moving media, such as the flowing water, are ubiquitous around us. This way, it is natural to ask whether the motion of positive-index media can provide an extra degree of freedom to enable the emergence of reversed Cherenkov radiation. Somehow, this open scientific question remains underexplored; particularly, whether the creation of reversed Cherenkov radiation has any fundamental restriction on the motion velocity of positive-index media remains elusive.

To address this issue, we find the possibility to create the reversed Cherenkov radiation from homogeneous positive-index moving media by exploiting the Fizeau–Fresnel drag effect. The underlying mechanism is that since the Fizeau–Fresnel drag effect is capable of dragging the emitted light toward the direction of moving media, the Fizeau–Fresnel drag effect provides the enticing possibility to drag the emitted light toward the reversed direction of moving charged particles. As background, the Fizeau–Fresnel drag effect[61–63] was first postulated by Fresnel in 1818,[64] experimentally verified by Fizeau in 1851[65] and later helped shape the theory of relativity[66,67] and facilitated the analysis of quantum friction.[68–71] Despite the long research history of free-electron radiation[72–81] and the Fizeau–Fresnel drag effect,[82–88] we highlight that the intrinsic connection between the reversed Cherenkov radiation and the Fizeau–Fresnel drag effect has never been explored before. Moreover, we discover that the reversed Cherenkov radiation may occur, only when the medium velocity v_{medium} exceeds a critical threshold c/n^2, namely, $v_{\mathrm{medium}} > c/n^2$. That is, the reversed Cherenkov radiation from positive-index moving media may emerge when the medium velocity is either subluminal (i.e., $c/n^2 < v_{\mathrm{medium}} < c/n$) or superluminal (i.e., $v_{\mathrm{medium}} > c/n$). Remarkably, under the superluminal scenario with $v_{\mathrm{medium}} > c/n$, we find the reversed Cherenkov radiation becomes intrinsically thresholdless for the velocity of moving charged particles.

Figure 1 shows the conceptual illustration of reversed Cherenkov radiation via Fizeau–Fresnel drag. The Fizeau–Fresnel drag effect can be vividly understood from the analysis of boat's motion on the flowing water in Figs. 1(a) and 1(b). Briefly speaking, while the boat moves toward the $-\hat{z}'$ direction under the water co-moving frame S' in Fig. 1(a), the boat may become to move toward the $+\hat{z}$ direction under the static river frame S in Fig. 1(b), due to the Fizeau–Fresnel drag

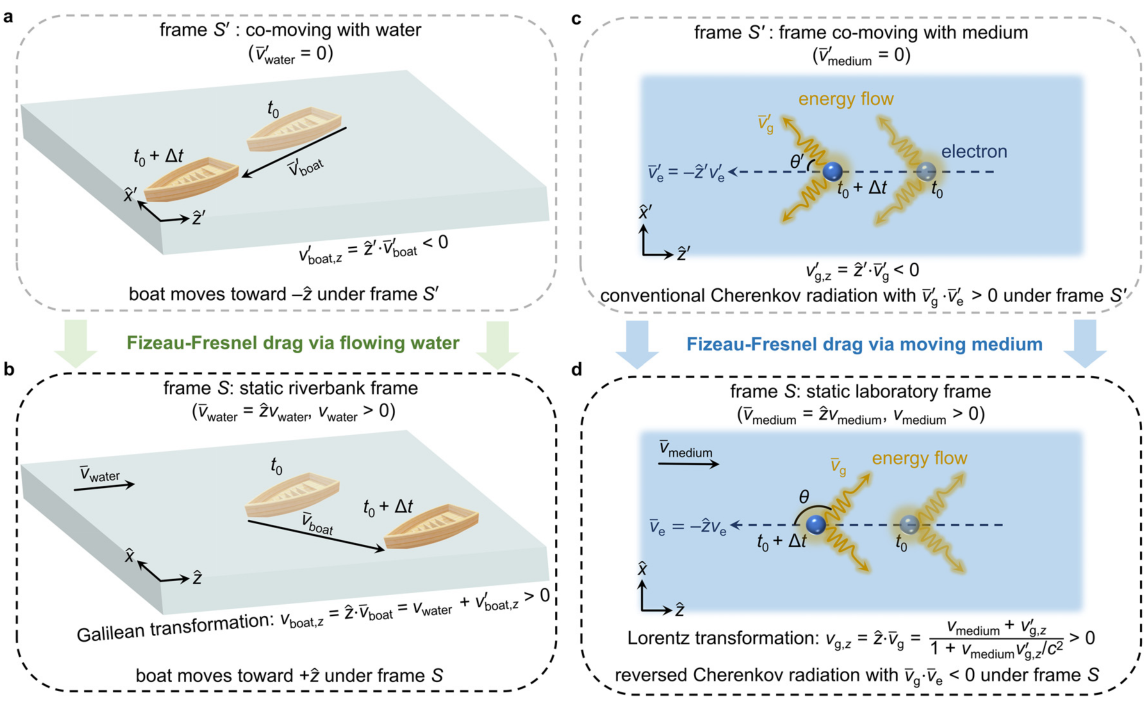

FIG. 1. Conceptual schematic of reversed Cherenkov radiation via Fizeau–Fresnel drag. (a) and (b) Fizeau–Fresnel drag of boat via flowing water. A boat moves with a velocity $\bar{v}'_{\mathrm{boat}}$ toward the $-\hat{z}'$ direction under the frame S' in (a), which is co-moving with water, namely, $v'_{\mathrm{boat},z} = \hat{z}' \cdot \bar{v}'_{\mathrm{boat}} < 0$. Under the static riverbank frame S in (b), the water moves with a velocity $\bar{v}_{\mathrm{water}} = \hat{z}v_{\mathrm{water}}$ toward the $+\hat{z}$ direction, and the boat can move with a velocity $\bar{v}_{\mathrm{boat}}$ toward the $+\hat{z}$ direction if $v_{\mathrm{boat},z} = \hat{z} \cdot \bar{v}_{\mathrm{boat}} = v_{\mathrm{water}} + v'_{\mathrm{boat},z} > 0$ (i.e., $v_{\mathrm{water}} > |v'_{\mathrm{boat},z}|$). (c) and (d) Fizeau–Fresnel drag of light via the moving medium. Under the frame S' co-moving with the medium in (c), an electron moves with a velocity $\bar{v}'_{\mathrm{e}} = -\hat{z}'v'_{\mathrm{e}}$, and the medium at rest has a positive refractive index n. Accordingly, the induced light emission has $\bar{v}'_{\mathrm{g}} \cdot \bar{v}'_{\mathrm{e}} > 0$ and corresponds to the conventional Cherenkov radiation, where $\bar{v}'_{\mathrm{g}}$ is the group velocity of light under the frame S'. Under the static laboratory frame S in (d), the medium moves with a velocity $\bar{v}_{\mathrm{medium}} = \hat{z}v_{\mathrm{medium}}$ toward the $+\hat{z}$ direction, and the electron has a velocity $\bar{v}_{\mathrm{e}} = -\hat{z}v_{\mathrm{e}}$. The light emission under the frame S can have $\bar{v}_{\mathrm{g}} \cdot \bar{v}_{\mathrm{e}} < 0$ and corresponds to the reversed Cherenkov radiation if $v_{\mathrm{g},z} = \frac{v_{\mathrm{medium}} + v'_{\mathrm{g},z}}{1 + v_{\mathrm{medium}}v'_{\mathrm{g},z}/c^2} > 0$ (i.e., $v_{\mathrm{medium}} > |v'_{\mathrm{g},z}|$), where $v'_{\mathrm{g},z} = \hat{z}' \cdot \bar{v}'_{\mathrm{g}}$ and $v_{\mathrm{g},z} = \hat{z} \cdot \bar{v}_{\mathrm{g}}$.

applied by the flowing water on the boat. Similarly, the moving medium can act like the flowing water and apply the Fizeau–Fresnel drag on the propagating light emitted by charged particles (e.g., free electrons used below), as schematically shown in Figs. 1(c) and 1(d). Without loss of generality, below we assume that under the static laboratory frame S in Fig. 1(d), the medium moves toward the $+\hat{z}$ direction with a velocity $\bar{v}_{\text{medium}} = \hat{z}v_{\text{medium}}$ (i.e., $v_{\text{medium}} > 0$), and the swift electron moves antiparallelly to the medium toward the $-\hat{z}$ direction with a velocity of $\bar{v}_e = -\hat{z}v_e$ (i.e., $v_e > 0$).

We begin with the analysis under the medium co-moving frame S' in Fig. 1(c). For conceptual illustration, we set that the static isotropic medium is nondispersive, lossless and has a constant positive refractive index n ($n > 1$). According to the Lorentz transformation, the electron moves with a velocity of $\bar{v}'_e = -\hat{z}'v'_e$, where $v'_e = \frac{v_e + v_{\text{medium}}}{1 + v_e v_{\text{medium}}/c^2}$. Since $v_{\text{medium}} > 0$ and $v_e > 0$, we have $v'_e > 0$. This way, under the frame S', the swift electron also moves toward the $-\hat{z}'$ direction. According to the Frank–Tamm formula, when $v'_e > c/n$, the emitted light generally has $\bar{v}'_g \cdot \bar{v}'_e > 0$ and $\cos\theta' = c/nv'_e$, where $|\bar{v}'_g| = c/n$ is the group velocity of emitted light under the frame S' and θ' is the angle between $\bar{v}'_g$ and $\bar{v}'_e$ (or $-\hat{z}'$). Accordingly, we have $\theta' = \arccos(c/nv'_e) < 90°$ and $v'_{g,z} = \hat{z}' \cdot \bar{v}'_g = -|\bar{v}'_g|\cos\theta' = -c/n \cdot c/nv'_e = -c^2/n^2 v'_e < 0$, as schematically shown in Fig. 1(c). Since $v'_{g,z} < 0$, the emitted light should also propagate toward the $-\hat{z}'$ direction. This scenario essentially corresponds to the emergence of conventional Cherenkov radiation under the medium co-moving frame S' in Fig. 1(c).

We now proceed to the analysis under the static laboratory frame S in Fig. 1(d). According to the Lorentz transformation, the group velocity $\bar{v}_g$ of emitted light under the frame S in Fig. 1(d) has

$$v_{g,z} = \hat{z} \cdot \bar{v}_g = \frac{v_{\text{medium}} + v'_{g,z}}{1 + v_{\text{medium}} v'_{g,z}/c^2}. \tag{1}$$

Upon close inspection of $v_{g,z}$ in Eq. (1), its denominator is always positive, namely, $1 + v_{\text{medium}} v'_{g,z}/c^2 > 0$, since $|v'_{g,z}| \le c$ and $v_{\text{medium}} \le c$ according to the relativity theory.

To enable the emergence of reversed Cherenkov radiation with $\theta > 90°$ under the frame S, the emitted light should propagate to the reversed direction of electron's motion, namely, the $+\hat{z}$ direction, where θ is the angle between $\bar{v}_g$ and $\bar{v}_e$ (or $-\hat{z}$) under the frame S. In other words, $v_{g,z} > 0$ should be satisfied. Since the denominator of $v_{g,z}$ in Eq. (1) is positive, this actually requires the numerator of $v_{g,z}$ in Eq. (1) to be positive, namely,

$$v_{\text{medium}} + v'_{g,z} = v_{\text{medium}} - |v'_{g,z}| > 0. \tag{2}$$

Since $|v'_{g,z}| = c^2/n^2 v'_e$, Eq. (2) can be re-organized to

$$v_{\text{medium}} > c^2/n^2 v'_e \quad \text{or} \quad v'_e > c^2/n^2 v_{\text{medium}}. \tag{3}$$

Moreover, since $v'_e \le c$, the necessary and sufficient condition to enable the emergence of reversed Cherenkov radiation can be obtained as

$$c \ge v'_e > c^2/n^2 v_{\text{medium}}. \tag{4}$$

From Eq. (4), we must always have

$$v_{\text{medium}} > c/n^2. \tag{5}$$

According to Eq. (5), the reversed Cherenkov radiation inside moving positive-index media may emerge only when the medium velocity v_{medium} exceeds the critical threshold $v_{\text{th,rCR,medium}}$, namely, $v_{\text{medium}} > v_{\text{th,rCR,medium}}$, where

$$v_{\text{th,rCR,medium}} = c/n^2. \tag{6}$$

The scaling factor $1/n^2$ could also be obtained via the geometric relation between the particle velocity and the group velocity of light, as shown in Fig. S5.

On the other hand, Eq. (5) indicates that the reversed Cherenkov radiation under the static laboratory frame S may emerge if the medium velocity is either superluminal ($v_{\text{medium}} > c/n$) or subluminal ($c/n^2 < v_{\text{medium}} < c/n$). For conceptual illustration, below $n = 2$ is used. To facilitate the understanding, the isofrequency contours of light inside the moving medium under the frame S, which are widely used in the analysis of quantum friction,[69] are plotted in Figs. 2 and 3. Specifically, these isofrequency contours of light inside the moving

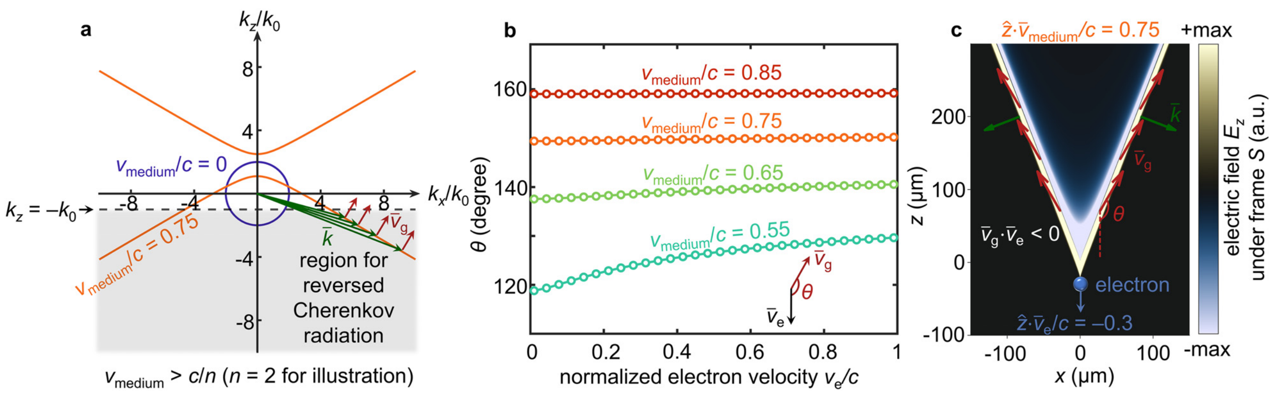

FIG. 2. Reversed Cherenkov radiation inside superluminal moving media. Here and below, we set the refractive index $n = 2$ for illustration. (a) Isofrequency contours of eigenmodes inside the moving medium. The dashed line corresponds to $k_z = -k_0$, where $k_0 = \omega/c$ is the wavevector of light in vacuum and ω is the angular frequency of emitted light. (b) Dependence of the radiation angle θ on the electron velocity v_e for various v_{medium}, where θ is the angle between the group velocity $\bar{v}_g$ of emitted light and $\bar{v}_e$ (or $-\hat{z}$). (c) Distribution of the electric field E_z induced by the swift electron, where $v_{\text{medium}}/c = 0.75$ and $v_e/c = 0.3$.

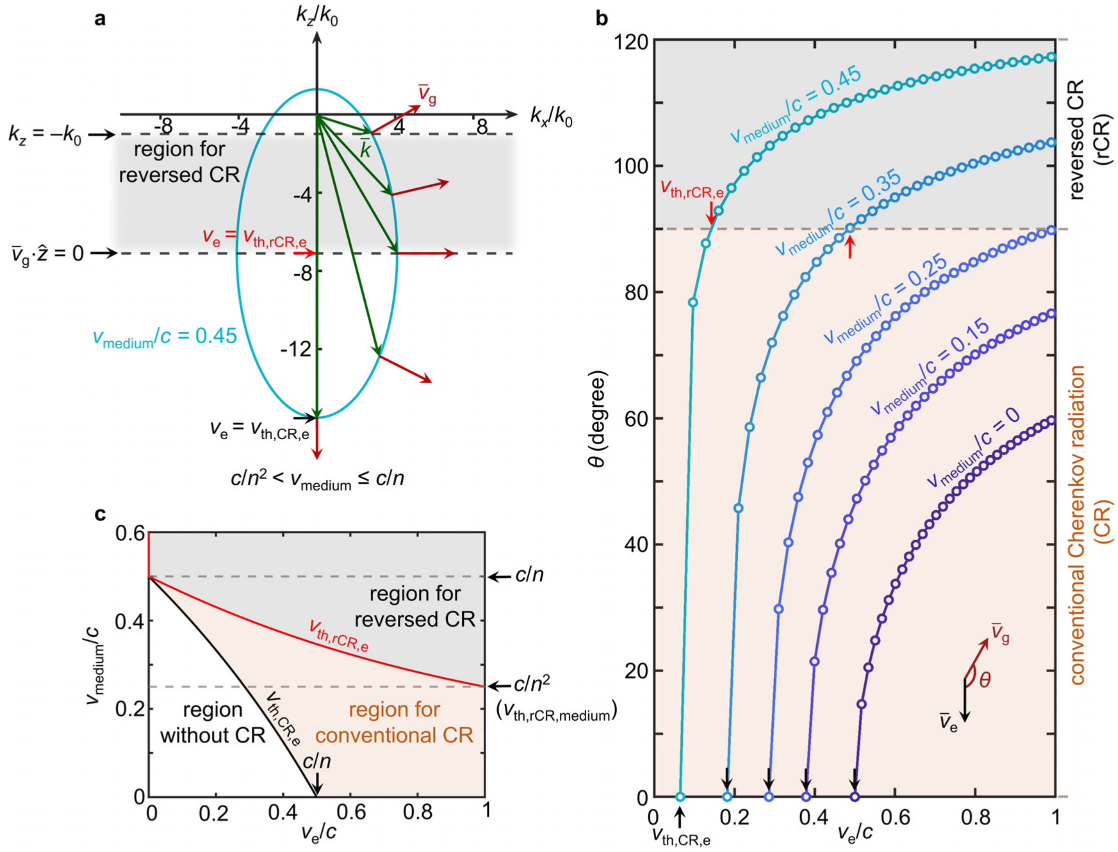

FIG. 3. Reversed Cherenkov radiation inside subluminal moving media. Under the laboratory frame S, $v_{th,CR,e}$($v_{th,rCR,e}$) corresponds to the threshold of electron velocity v_e for the conventional (reversed) Cherenkov radiation, and $v_{th,rCR,medium} = c/n^2$ corresponds to the threshold of medium velocity v_{medium} to enable the emergence of reversed Cherenkov radiation. (a) Isofrequency contour of eigenmodes of light inside the moving medium with $v_{meidum}/c = 0.45$. (b) Dependence of the radiation angle θ on the electron velocity v_e under various v_{medium}. (c) Phase diagram of Cherenkov radiation in the $v_e - v_{medium}$ parameter space.

medium under the laboratory frame S are governed by $k_x^2 + \frac{c^2-n^2v_{medium}^2}{c^2-v_{medium}^2}\left[k_z - \frac{nc+v_{medium}}{nv_{medium}+c}\frac{\omega}{c}\right]\cdot\left[k_z - \frac{nc-v_{medium}}{nv_{medium}-c}\frac{\omega}{c}\right] = 0$,[42] where ω and $\bar{k} = \hat{x}k_x + \hat{z}k_z$ are the angular frequency and the wavevector of light, respectively. From this equation, both the symmetry center and shape of the isofrequency contour essentially vary with the medium velocity v_{medium}. For example, distinct hyperbolic or elliptical isofrequency contours emerge in Figs. 2(a) and 3(a), which owes to the fact that the response tensors of isotropic media in motion are the same as that of stationary bianisotropic media.[42]

Figure 2 shows the superluminal scenario with $v_{medium} > c/n$. Since the swift electron and the medium move toward two reversed directions, the electron velocity $v'_e = \frac{v_e+v_{medium}}{1+v_e v_{medium}/c^2}$ under the medium co-moving frame S' would increase monotonically with v_e. This way, we have the following inequality:

$$v'_e \ge \min\left(\frac{v_e + v_{medium}}{1 + v_e v_{medium}/c^2}\right) = \frac{v_e + v_{medium}}{1 + v_e v_{medium}/c^2}\Big|_{v_e = 0} = v_{medium}. \tag{7}$$

Since $v_{medium} > c/n$, Eq. (7) can be transformed into

$$v'_e > c/n \quad \text{for} \quad \forall v_e \in [0, c], \quad \text{if} \quad v_{medium} > c/n. \tag{8}$$

Moreover, since $c/n = c/n \cdot \frac{c/n}{c/n} > c/n \cdot \frac{c/n}{v_{medium}} = c^2/n^2 v_{medium}$, Eq. (8) can be re-written as

$$v'_e > c^2/n^2 v_{medium} \quad \text{for} \quad \forall v_e \in [0, c], \quad \text{if} \quad v_{medium} > c/n. \tag{9}$$

Equation (9) exactly corresponds to the necessary and sufficient condition to enable the emergence of reversed Cherenkov radiation in Eq. (3). Essentially, Eq. (9) indicates that under the static laboratory frame S, when the medium moves superluminally with $v_{medium} > c/n$, the reversed Cherenkov radiation can occur for arbitrary electron velocity v_e and is thus thresholdless for the electron velocity.

This thresholdless feature can be further verified through the isofrequency contour analysis of eigenmodes of light in moving media in Fig. 2(a). By enforcing the momentum-matching condition between the swift electron and the emitted light along the $\hat{z}$ direction, the wavevector of emitted light under the frame S has $k_z = \hat{z} \cdot \bar{k} = -\omega/v_e$. Since $0 \le v_e \le c$, these emitted light always has $k_z \le -\omega/c$. For eigenmodes with $k_z \le -\omega/c$ in Fig. 2(a), their group velocity always has $v_{g,z} > 0$. Under this scenario, the emitted light always has $\bar{v}_g \cdot \bar{v}_e = (\hat{z}v_{g,z}) \cdot (-\hat{z}v_e) < 0$ and thus $\theta > 90°$ in Figs. 2(b) and 2(c) for arbitrary electron velocity v_e, indicating the occurrence of thresholdless reversed Cherenkov radiation under the frame S.

Figures 3 and 4 show the subluminal scenario with $c/n^2 < v_{\text{medium}} < c/n$. Under this scenario, we find the existence of electron velocity threshold $v_{\text{th,rCR,e}}$ for the reversed Cherenkov radiation in Figs. 3(a)–3(c), which is dependent on the medium velocity v_{medium}. Recall the two facts that the necessary and sufficient condition to induce the reversed Cherenkov radiation is $v'_{\text{e}} > c^2/n^2 v_{\text{medium}}$, as governed by Eq. (3), and meanwhile, that the electron velocity $v_{\text{e}} = \frac{v'_{\text{e}} - v_{\text{medium}}}{1 - v'_{\text{e}} v_{\text{medium}}/c^2}$ under the frame S increases monotonically with v'_{e}. This way, the reversed Cherenkov radiation under the frame S would emerge only if $v_{\text{e}} > v_{\text{th,rCR,e}}$ (see case 1 in Fig. 4), where

$$v_{\text{th,rCR,e}} = \left.\frac{v'_{\text{e}} - v_{\text{medium}}}{1 - v'_{\text{e}} v_{\text{medium}}/c^2}\right|_{v'_{\text{e}} = c^2/n^2 v_{\text{medium}}} = \frac{1/n^2 - v^2_{\text{medium}}/c^2}{(1 - 1/n^2) v_{\text{medium}}/c} c. \tag{10}$$

According to Eq. (10), $v_{\text{th,rCR,e}}$ would decrease from c to 0 when the medium velocity v_{medium} increases from c/n^2 to c/n, as shown in Figs. 3(b) and 3(c). This is consistent with the isofrequency contour analysis in Fig. 3(a), where the emitted light would have $v_{\text{g},z} > 0$ if $-\omega/v_{\text{th,rCR,e}} < k_z = -\omega/v_{\text{e}} \le -\omega/c$ (i.e., $v_{\text{th,rCR,e}} < v_{\text{e}} \le c$).

Similarly, we also find the existence of electron velocity threshold $v_{\text{th,CR,e}}$ for conventional Cherenkov radiation, since the necessary and sufficient condition for its emergence requires $v'_{\text{e}} > c/n$. This way, the conventional Cherenkov radiation under the frame S would emerge only if $v_{\text{e}} > v_{\text{th,CR,e}}$ (see cases 2 and 3 in Fig. 4), where

$$v_{\text{th,CR,e}} = \left.\frac{v'_{\text{e}} - v_{\text{medium}}}{1 - v'_{\text{e}} v_{\text{medium}}/c^2}\right|_{v'_{\text{e}} = c/n} = \frac{1/n - v_{\text{medium}}/c}{1 - 1/n \cdot v_{\text{medium}}/c} c. \tag{11}$$

According to Eq. (11), $v_{\text{th,CR,e}}$ would decrease from c/n to 0 when the medium velocity v_{medium} increases from 0 to c/n in Figs. 3(b) and 3(c).

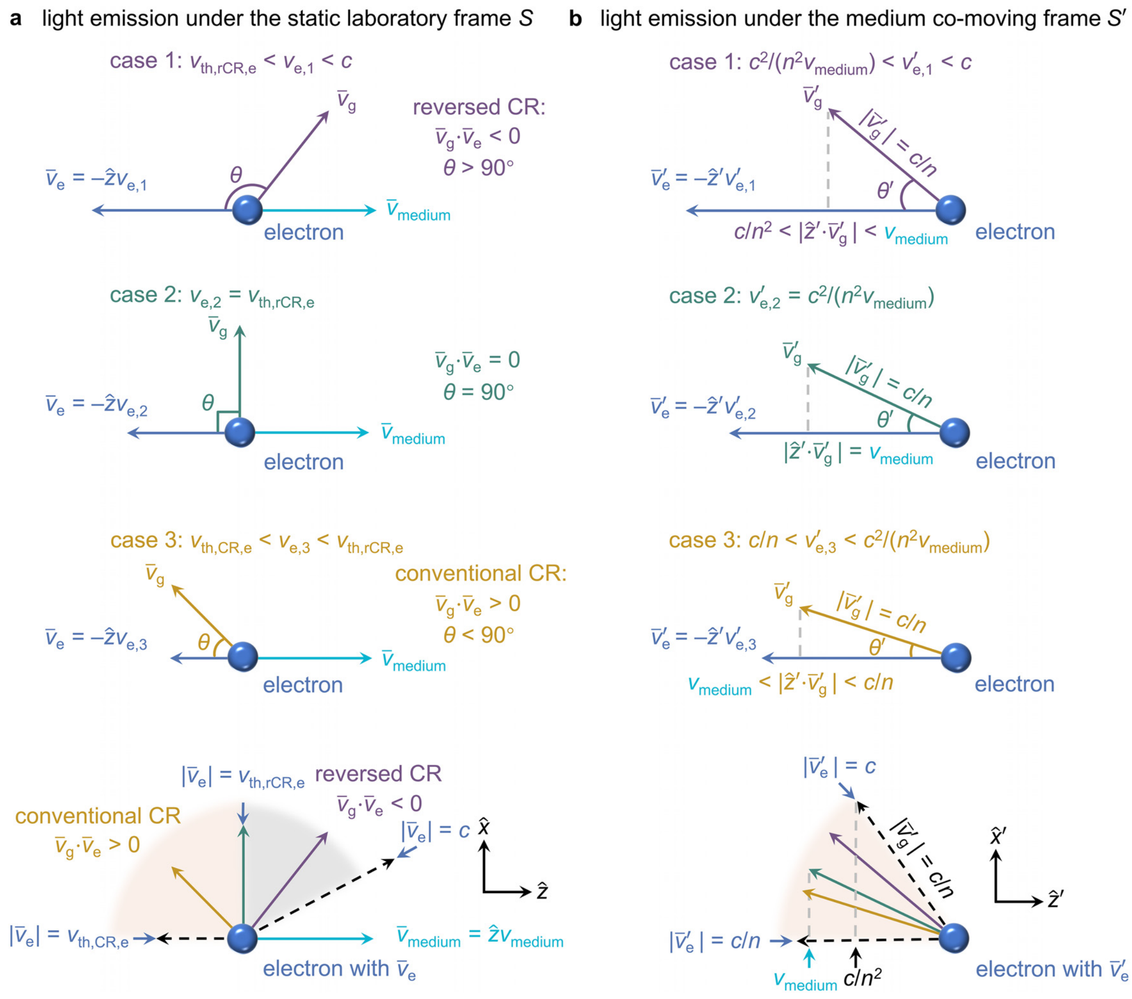

FIG. 4. Underlying mechanism of reversed Cherenkov radiation inside subluminal moving media with $c/n^2 < v_{\text{medium}} < c/n$ under the laboratory frame S. (a) Light emission under the static laboratory frame S. For case 1 with $v_{\text{th,rCR,e}} < v_{\text{e}} = v_{\text{e},1} < c$, the light emission under the frame S has $\bar{v}_{\text{g}} \cdot \bar{v}_{\text{e}} < 0$ and $\theta > 90°$, which corresponds to the reversed Cherenkov radiation. For case 2 with $v_{\text{e}} = v_{\text{e},2} = v_{\text{th,rCR,e}}$, the light emission under the frame S has $\bar{v}_{\text{g}} \cdot \bar{v}_{\text{e}} = 0$ and $\theta = 90°$. For case 3 with $v_{\text{th,CR,e}} < v_{\text{e}} = v_{\text{e},3} < v_{\text{th,rCR,e}}$, the light emission under the frame S has $\bar{v}_{\text{g}} \cdot \bar{v}_{\text{e}} > 0$ and $\theta < 90°$, which corresponds to the conventional Cherenkov radiation. (b) Light emission under the medium co-moving frame S'. For comparison, each case under the frame S in (a) is correspondingly re-plotted under the frame S' in (b).

Under this scenario, the emitted light always has $v_{g,z} < 0$ if $-\omega/v_{\mathrm{th,CR,e}} < k_z = -\omega/v_e < -\omega/v_{\mathrm{th,rCR,e}}$ (i.e., $v_{\mathrm{th,CR,e}} < v_e < v_{\mathrm{th,rCR,e}}$), as shown in Fig. 3(a).

In conclusion, we have revealed the possibility to create the reversed Cherenkov radiation from positive-index moving media by exploiting the Fizeau–Fresnel drag effect. Remarkably, we have discovered the existence of medium velocity threshold for the creation of reversed Cherenkov radiation, namely, $v_{\mathrm{th,rCR,medium}} = c/n^2$, which is solely determined by the medium's refractive index. Specifically, we have further found that the reversed Cherenkov radiation becomes thresholdless for the charged particle velocity when the medium moves superluminally. We highlight that the moving media, in principle, could be effectively constructed, for example, by using spatiotemporal materials,[89–101] nonreciprocal graphene or metal biased with a drift current,[102–109] and nonlinear materials under the illumination of laser pulses.[110–112] Due to the rich un-explored physics in the realm of light–particle–matter interactions, our finding may spark the continuous exploration of free-electron radiation (including reversed Cherenkov radiation) in more complex yet exotic photonic systems, such as spatiotemporal materials, nonreciprocal metasurfaces, and nonlinear materials.

See the supplementary material for five sections, including derivation of Cherenkov radiation under the medium co-moving frame S', derivation of Cherenkov radiation under the static laboratory frame S, derivation of dispersion relation for moving isotropic media, derivation of the radiation angle θ under the static laboratory frame S, and more discussion on the reversed Cherenkov radiation via Fizeau–Fresnel drag.

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AUTHOR DECLARATIONS

Conflict of Interest

The authors have no conflicts to disclose.

Author Contributions

Bowen Zhang and Zheng Gong contributed equally to this work.

Bowen Zhang: Conceptualization (equal); Investigation (equal); Visualization (equal); Writing – original draft (equal). **Zheng Gong:** Investigation (equal); Visualization (equal); Writing – review & editing (equal). **Ruoxi Chen:** Visualization (equal); Writing – review & editing (equal). **Xuhuinan Chen:** Visualization (equal); Writing – review & editing (equal). **Yi Yang:** Visualization (equal); Writing – review & editing (equal). **Hongsheng Chen:** Supervision (equal); Visualization (equal); Writing – review & editing (equal). **Ido Kaminer:** Supervision (equal); Visualization (equal); Writing – review & editing (equal). **Xiao Lin:** Conceptualization (equal); Supervision (equal); Visualization (equal); Writing – original draft (equal).

DATA AVAILABILITY

The data that support the findings of this study are available within the article and its supplementary material.

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