SUPPLEMENTAL MATERIAL

Here, we provide the derivation of Eqs. (10) and (11). We start with the probability $P(+)$ to post-select an electron in the $|+\rangle$ state, given an initial thermal cavity state $\rho_{\bar{n}}$, and calculate it as the trace of the following submatrix:

$$P(+) = \mathrm{Tr}\left(D_+(g_Q)\rho_{\bar{n}}D_+^\dagger(g_Q)\right) = \frac{1}{2} + \frac{1}{4}\frac{1}{\bar{n}+1}\sum_{m=0}^{\infty}\left(\frac{\bar{n}}{\bar{n}+1}\right)^m \langle m|D(g_Q) + D(-g_Q)|m\rangle. \tag{S1}$$

To find the matrix elements, we use results from [9]:

$$\langle m|D(g_Q)|m\rangle = e^{-\frac{|g_Q|^2}{2}}\sum_{m'=0}^{m}\binom{m}{m'}\frac{(-1)^{m'}}{m'!}|g_Q|^{2m'}, \tag{S2}$$

which yields

$$P(+) = \frac{1}{2} + \frac{e^{-\frac{|g_Q|^2}{2}}}{2(\bar{n}+1)}\sum_{m,m'}\binom{m}{m'}\left(\frac{\bar{n}}{\bar{n}+1}\right)^m\frac{(-|g_Q|^2)^{m'}}{m'!}. \tag{S3}$$

By rearranging the sums, we find the simplified form:

$$P(+) = \frac{1}{2}\left(1 + e^{-|g_Q|^2(\bar{n}+\frac{1}{2})}\right). \tag{S4}$$

The flowchart in Fig. S1 summarizes the cooling protocol, based on repeated applications of the electron–cavity interaction. The cavity is initially in a thermal state $\rho_{\bar{n}^{(0)}}$ with mean photon number $\bar{n}^{(0)}$. An electron is prepared in the superposition state $|+\rangle = (|0\rangle + |1\rangle)/\sqrt{2}$ using a beam splitter that creates the coherent superposition of two trajectories. The electron subsequently interacts with the cavity mode according to Eq. (5). Post-selection is then performed by measuring the electron state in the $|\pm\rangle$ basis. A successful outcome, corresponding to the $|+\rangle$ result, induces cooling on the cavity state. If the target temperature is not yet reached, another electron is created, with the interaction parameter multiplied by a factor of i relative to the previous cycle, which can be created by a quarter-cycle delay. This process is iterated until the desired degree of cooling is achieved. Below, we provide a more detailed analysis of the Oscillator Cooling Block (OCB), defined as a sequence of four Conditional Displacement (CD) operators with the interaction parameter cycling through the pattern $g_Q \to ig_Q \to -g_Q \to -ig_Q$.

To cool down the cavity state, we use the following combination of CD operators:

$$\mathrm{CD}\left(-ig_Q\right)\mathrm{CD}\left(-g_Q\right)\mathrm{CD}\left(ig_Q\right)\mathrm{CD}\left(g_Q\right),$$

where the CD operator is defined according to Eq. (5). To demonstrate the possibility of cooling down the cavity

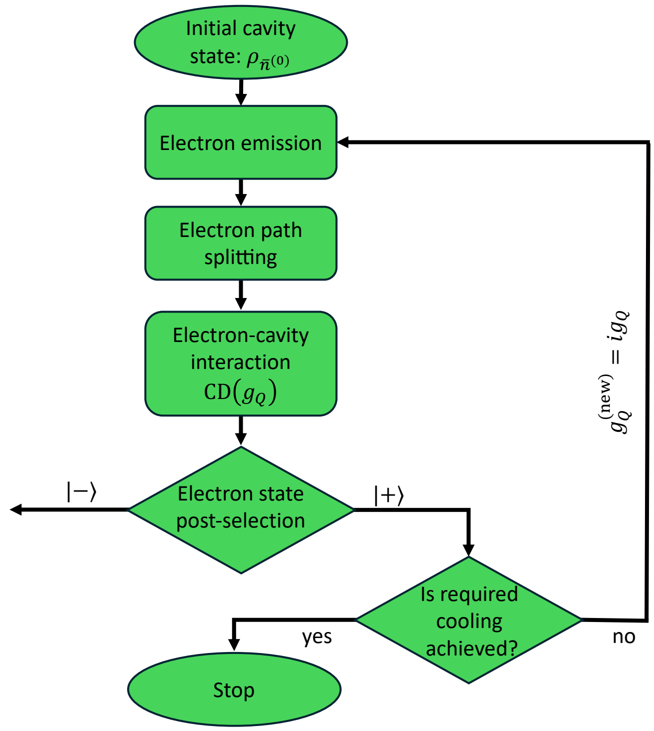

FIG. S1: **Flowchart of the cavity-cooling protocol based on electron–cavity interaction through the Conditional Displacement operator** CD(g_Q)**.** The process begins with the cavity in an initial thermal state. An electron is emitted and its trajectory is coherently split into two paths. It then interacts with the cavity via the CD operator. After the interaction, the electron is measured in the $|\pm\rangle$ basis. A measurement outcome $|-\rangle$ corresponds to a failed cooling attempt, whereas a $|+\rangle$ outcome contributes to cooling. The cavity state is subsequently checked against the target photon number: if the desired cooling level is reached, the protocol terminates; otherwise, a new electron is emitted and the interaction is repeated with the parameter updated as $g_Q \to ig_Q$. This loop continues until the required degree of cooling is achieved.

using the OCB, we consider below the case $|g_Q| \ll 1$. Up to the first non-trivial order, the corresponding OCB Kraus operator product (7) is expressed as

$$D_{\mathrm{OCB}}(g_Q) = 1 - \frac{1}{2}|g_Q|^2\left(2a^\dagger a + 1 - i\right) + O\left(|g_Q|^4\right). \tag{S5}$$

Thus, for small $|g_Q|$, the action of this operator on an arbitrary initial state leads to a reduction in the photon number. Specifically, if $|\psi_0\rangle$ is the initial state and

$$|\psi\rangle = \sqrt{\mathcal{N}}D_{\mathrm{OCB}}(g_Q)|\psi_0\rangle$$

is the resulting normalized state (with $\mathcal{N} > 0$ denoting the normalization constant), their photon numbers are related by

$$\langle\psi| a^\dagger a |\psi\rangle = \langle\psi_0| a^\dagger a |\psi_0\rangle - \\ -2\left|g_Q\right|^2 \left(\langle\psi_0| \left(a^\dagger a\right)^2 |\psi_0\rangle - \langle\psi_0| a^\dagger a |\psi_0\rangle^2\right) + O\left(\left|g_Q\right|^4\right). \tag{S6}$$

The probability to post-select four electrons in the $|+\rangle$ state can be calculated as

$$P = \frac{1}{\mathcal{N}} = 1 - \left|g_Q\right|^2 \left(2\langle\psi_0| a^\dagger a |\psi_0\rangle + 1\right) + O\left(\left|g_Q\right|^4\right). \tag{S7}$$

The same results can be obtained if the initial state is a mixed one.

Figure S2 illustrates the transformation of a coherent state $|\alpha\rangle$ under the action of D_{OCB}, where the shift of the Wigner function toward the origin reflects a reduction in photon number. Interestingly, the D_{OCB} cooling approach is general (applying for arbitrary states) but not optimal: for states with a well-defined phase, such as coherent states, a straight-forward coherent displacement provides a more efficient way of reducing the photon number.

Below, we explicitly consider the case where the initial state is thermal. Although the photon number variation and the cooling probability can be obtained from Eqs. (S6, S7) – with the averaging over the wave function replaced by averaging over the density matrix – the thermal state has an additional important property with respect to the OCB operation: it remains thermal up to corrections of order $\left|g_Q\right|^2$. To demonstrate this, we derive the corresponding expressions below.

Applying the operator (S5) to the thermal state (1) $\rho_{\bar{n}^{(0)}}$ with thermal photon number $\bar{n}^{(0)}$, we get the density matrix after one OCB:

$$\rho^{(1)} = \mathcal{N}\sum_{n=0}^{\infty}\left(1 - \left|g_Q\right|^2 (2n+1)\right)\left(\rho_{\bar{n}^{(0)}}\right)_{nn} |n\rangle\langle n| \\ + O\left(\left|g_Q\right|^4\right), \tag{S8}$$

where $\left(\rho_{\bar{n}^{(0)}}\right)_{nn}$ is the n-th diagonal element of $\rho_{\bar{n}^{(0)}}$. The normalization factor $\mathcal{N}$ is defined from the identity $\mathrm{Tr}\,\rho^{(1)} = 1$ and is therefore given by

$$\mathcal{N} = \frac{1}{1 - \left|g_Q\right|^2 \left(2\bar{n}^{(0)} + 1\right)} + O\left(\left|g_Q\right|^4\right). \tag{S9}$$

The thermal photon number after one OCB and post-selection is $\bar{n}^{(1)} = \mathrm{Tr}\left(a^\dagger a \rho^{(1)}\right)$, which up to order $\left|g_Q\right|^2$ equals

$$\bar{n}^{(1)} = \bar{n}^{(0)} \cdot \left(1 - 2\left|g_Q\right|^2 \left(\bar{n}^{(0)} + 1\right)\right) + O\left(\left|g_Q\right|^4\right). \tag{S10}$$

Within the same level of accuracy, Eq. (S8) coincides with the expression for thermal state $\rho_{\bar{n}^{(1)}}$ with the photon number $\bar{n}^{(1)}$. Thus, up to order $\left|g_Q\right|^2$, the state remains thermal, with its temperature decrease depending both on the interaction parameter g_Q and on the photon number $\bar{n}^{(0)}$.

Figure S3 illustrates how the Wigner function of a thermal state is modified by the OCB operation. Panel (a) shows the Wigner function of the initial thermal state $\rho_{\bar{n}^{(0)}}$. The action of $D_+(g_Q)$ with real positive g_Q is indicated by the green arrows. The resulting Wigner function, shown in panel (b), is displaced along the x direction and squeezed in p. The remaining components of the OCB – $D_+(ig_Q)$, $D_+(-g_Q)$, and $D_+(-ig_Q)$ – mutually compensate their respective displacements and collectively squeeze the state along both quadratures, as shown in panel (c). This combined action narrows the Wigner distribution, corresponding to a decrease in the effective temperature (see Eq. (2) in the main text).

The probability of post-selecting all four electrons in the $|+\rangle$ state is given by

$$P^{(1)} = \mathrm{Tr}\left(D_{\mathrm{OCB}}(g_Q)\rho_{\bar{n}^{(0)}} D^\dagger_{\mathrm{OCB}}(g_Q)\right) = \frac{1}{\mathcal{N}} \\ = 1 - \left|g_Q\right|^2 \left(2\bar{n}^{(0)} + 1\right) + O\left(\left|g_Q\right|^4\right). \tag{S11}$$

After k repetitions of the cooling procedure, the photon number and probability can be obtained by multiple recursive application of Eqs. (S10, S11). To gain intuition, we examine the limit of $k \ll \left|g_Q\right|^{-2}$, for which a simplified analytical result arises as the temperature does not decrease significantly during the cooling. In this case, one can neglect variations in the temperature factor inside the brackets and obtain:

$$\bar{n}^{(k)} \approx \bar{n}^{(0)} \cdot \left(1 - 2\left|g_Q\right|^2 \left(\bar{n}^{(0)} + 1\right)\right)^k \\ \approx \bar{n}^{(0)} \cdot \left(1 - 2k\left|g_Q\right|^2 \left(\bar{n}^{(0)} + 1\right)\right), \tag{S12}$$

$$P^{(k)} \approx \left(1 - \left|g_Q\right|^2 \left(2\bar{n}^{(0)} + 1\right)\right)^k \\ \approx 1 - k\left|g_Q\right|^2 \left(2\bar{n}^{(0)} + 1\right). \tag{S13}$$

Equations (S12, S13) hold in the absence of thermal exchange with the environment. In the presence of a thermal bath ($\kappa \neq 0$), however, the cooling dynamics changes significantly. After repeated applications of OCB, the system enters a stable regime where the photon number $\bar{n}(t)$ oscillates near the final value $\bar{n}_{\mathrm{f}}$. It is therefore useful to estimate the magnitude of these oscillations. Assuming that the stable state closely approximates a thermal state with photon number $\bar{n}_{\mathrm{f}}$ (a reasonable approximation, as our results show fidelity 99% or higher) the photon number oscillations magnitude $\delta\bar{n}$ can be estimated using the Lindblad equation:

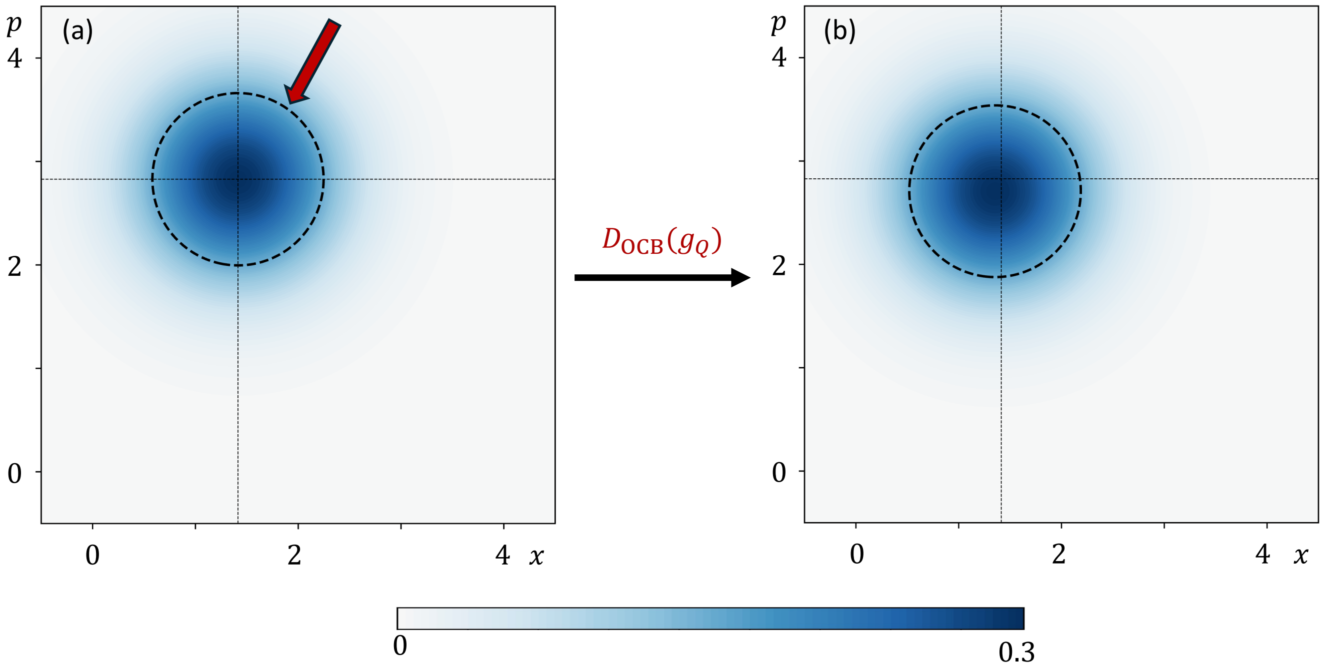

FIG. S2: **Shift of a coherent state induced by the Oscillator Cooling Block (OCB).** (a) Wigner function of the initial coherent state $|\alpha\rangle$. (b) Wigner function of the state after application of $D_{\mathrm{OCB}}(g_Q)$. The OCB shifts the Wigner function toward the origin in phase space (red arrow), thereby reducing the photon number. The dashed contour line represents the contour at half of the maximum value. The vertical and horizontal dotted lines correspond to the center of the initial state: $x = \sqrt{2}\Re\alpha$, $p = \sqrt{2}\Im\alpha$. Parameters are $\alpha = 1 + 2i$ and $g_Q = 0.25$.

$$\delta\bar{n} \approx \frac{\partial \bar{n}(t)}{\partial t}\delta t = \mathrm{Tr}\left(a^\dagger a \dot{\rho}\right)\delta t = \mathrm{Tr}\left(a^\dagger a\left(\frac{\kappa}{2}\left(n_{\mathrm{b}}+1\right)\left(2a\rho a^\dagger - a^\dagger a\rho - \rho a^\dagger a\right) + \frac{\kappa}{2}n_{\mathrm{b}}\left(2a^\dagger\rho a - aa^\dagger\rho - \rho aa^\dagger\right)\right)\right)\delta t \approx$$
$$\approx \mathrm{Tr}\left(a^\dagger a\left(\frac{\kappa}{2}\left(n_{\mathrm{b}}+1\right)\left(2a\rho_{\bar{n}_{\mathrm{f}}}a^\dagger - a^\dagger a\rho_{\bar{n}_{\mathrm{f}}} - \rho_{\bar{n}_{\mathrm{f}}}a^\dagger a\right) + \frac{\kappa}{2}n_{\mathrm{b}}\left(2a^\dagger\rho_{\bar{n}_{\mathrm{f}}}a - aa^\dagger\rho_{\bar{n}_{\mathrm{f}}} - \rho_{\bar{n}_{\mathrm{f}}}aa^\dagger\right)\right)\right)\delta t = \kappa\delta t\left(n_{\mathrm{b}} - \bar{n}_{\mathrm{f}}\right). \tag{S14}$$

When substituting ρ with $\rho_{\bar{n}_{\mathrm{f}}}$, we rely on our numerical results showing that the stable state reached after a sufficiently large number of cooling rounds closely approximates the thermal state $\rho_{\bar{n}_{\mathrm{f}}}$, with a fidelity of 99% or higher.

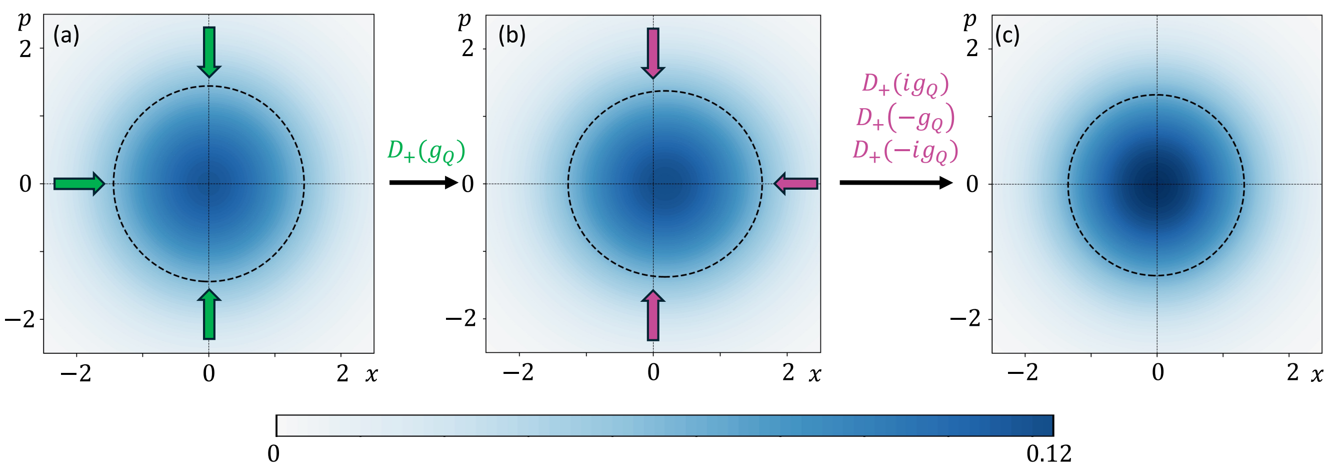

FIG. S3: **Variation of a thermal state under the action of the Oscillator Cooling Block (OCB).** (a) Wigner function of the initial thermal state $\rho_{\bar{n}^{(0)}}$ with mean photon number $\bar{n}^{(0)} = 1$. (b) Action of the Kraus operator $D_{\rm OCB}(g_Q)$ with $g_Q = 0.25$ squeezes the state along the p-axis and shifts it along the x-axis. (c) Applying the full OCB combination compensates the shifts and produces squeezing along both quadratures. Dashed contour lines indicate the half-maximum level.