annalen der physik
www.ann-phys.org

Spin-Spacetime Censorship

*Jonathan Nemirovsky, Eliahu Cohen, and Ido Kaminer**

Quantum entanglement and relativistic causality are key concepts in theoretical works seeking to unify quantum mechanics and gravity. In this article, a gedanken experiment that couples the spin to spacetime is proposed, and is then analyzed in the context of quantum information by using different approaches to quantum gravity. Both classical gravity theory and certain quantum theories predict that around a spin-half particle, the spherical symmetry of spacetime is broken by its magnetic field or merely by its intrinsic angular momentum. It is asserted that any spin-related deviation from spherical symmetry, upon appropriate measurement, can violate relativistic causality and quantum no-cloning. To avoid these violations, the measurable spacetime around the particle's rest frame shall typically remain spherically symmetric, potentially as a back-action by the act of a covariant measurement, or due to a quantized spin-dependence of the magnetic field. This way, this gedanken experiment suggests a censorship mechanism preventing the possibility of spacetime-based spin detection, which can shed light on the interface between quantum mechanics and gravity. Since this proposed gedanken experiment is independent of any specific theory, it is suitable for testing the coupling of quantum matter and spacetime in present and future candidate theories of quantum gravity.

1. Introduction

In 1915, general relativity revolutionized our view and understanding of the universe. Black holes, gravity waves, gravitational time dilation, gravitational lensing, and gravitational redshift are just a few amazing examples of its predictive power. Yet even today, and although several promising approaches have been proposed,[1–4] it is still unclear how to reconcile general relativity with the theory of quantum mechanics. This apparent incompatibility has been accentuated by the information paradox in black holes[5] and by the AMPS paradox.[6] These paradoxes, however, do not necessarily indicate whether and how gravity can be unified with quantum mechanics.

The necessity of unifying gravity and quantum mechanics has been the subject of much interest and important debates from Feynman's 1957 gedanken experiment,[7] through many analyses over the years,[8–15] and even very recently.[16–24] Particularly related to the current paper are works suggesting experiments on the interface between gravity and quantum information[18,19,21,22] that could prove the necessity of a quantum theory of gravity. Such experiments may remove doubts[25,26] regarding the empirical testability of the would-be unified theory. Our aim in this work is different, as we focus on the question of how to consistently couple quantum spins with spacetime, while maintaining relativistic causality. Such a consistent theory must have a specific mechanism that ensures relativistic causality. This mechanism is expected to provide new insight regarding the way in which quantum matter generates spacetime curvature. This is currently an interesting, open question, especially when one tries to quantize spacetime. Our conclusions apply to studies regarding semiclassical gravity, linearized quantum gravity, and standard usages of the ADM formalism, as well as to works in quantum foundations regarding nonlinearity and gravitational decoherence.

Below, we present a new gedanken experiment that may shed new light on possible paths toward a unified theory of quantum mechanics and gravity. Specifically, our gedanken experiment can be used today, without waiting for an experimental realization. Prospectively, it can serve as a testing ground for theoretical models attempting to describe quantum measurements of spacetime, quantized or not.

While formulating a unified theory of quantum gravity is a major challenge, even the much simpler question of how to maintain relativistic causality in the quantum world has led to important "no-go" theorems in quantum information, for example, the "no-signaling" principle[27] or its successors, the "no-communication" theorem,[28] which forbids instantaneous transfer of information between two observers, as well as the "no-cloning" theorem[29,30] or the "no teleportation" theorem (see, e.g., ref. [31]) which forbid the creation of an identical copy of an arbitrary unknown quantum state. Our goal is to show how such considerations, coming from the field of quantum information, may give rise to new insights regarding the sought-after description of quantum measurements in quantum gravity.

Our work presents a gedanken experiment that tests how a spin is coupled to the (either classical or quantum) spacetime

J. Nemirovsky, I. Kaminer
Technion – Israel Institute of Technology
Haifa 32000, Israel
E-mail: kaminer@technion.ac.il
E. Cohen
Bar Ilan University
Ramat Gan 5290002, Israel

The ORCID identification number(s) for the author(s) of this article can be found under https://doi.org/10.1002/andp.202100348

DOI: 10.1002/andp.202100348

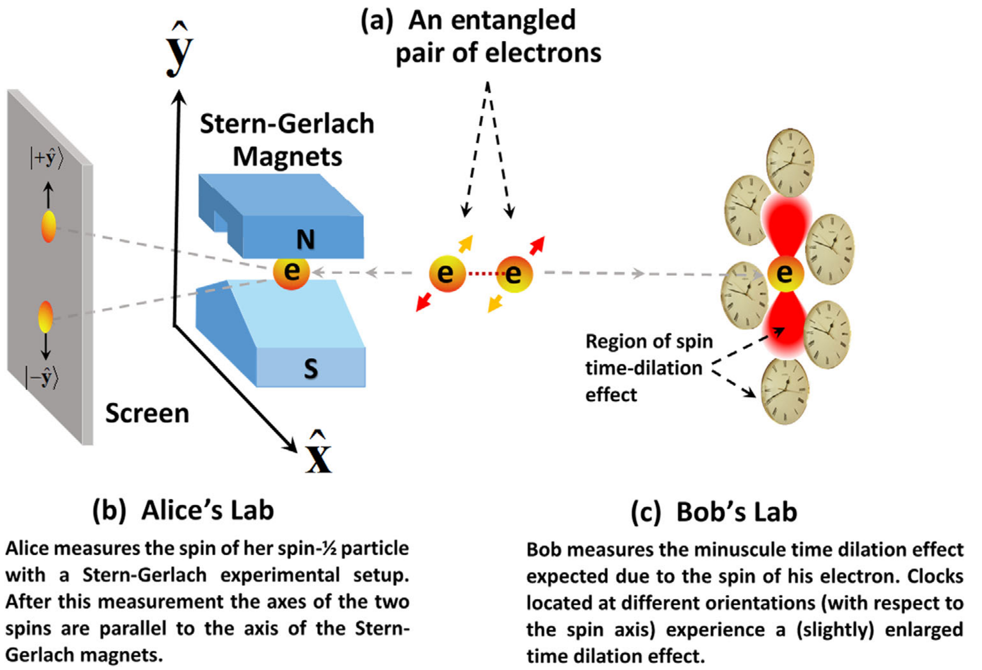

Figure 1. The gedanken experiment. The gedanken experiment shows that the measurable spacetime around a spin-½ particle (marked by 'e') cannot show symmetry breaking correlated to the spin axis, or else information can be transmitted at a superluminal speed. The stages of the experiments are as follows: a) A pair of entangled spin-½ particles is prepared and distributed to Alice and Bob. b) Alice performs an ordinary Stern–Gerlach quantum measurement on her particle's spin. She decides to orient her magnets' axes parallel to the $\hat{x}$ axis or parallel to the $\hat{y}$ axis. The spin of Bob's particle is then also parallel to the $\hat{x}$ or $\hat{y}$ axis (depending on, and in correlation with, the magnets' axis that Alice chose). c) Bob places clocks around the spin and measures the hands of the clocks to infer the time dilation around the particle. Any break of spherical symmetry in the reading of the clocks will allow Bob to determine the axis of the spin and find out how Alice oriented her magnets, even if he is outside of her light cone. This should be impossible because it contradicts causality. It therefore follows that the measurable time dilation must be spherically symmetric, potentially as a back-action by the mere act of measurement, thus censoring the spin axis measurement.

around it and what should various measurements of the spacetime tell us about the spin. This coupling is measured with clocks that are arranged symmetrically around the spin. The hands of the clock show the time dilation of each clock, which can be used to infer the axis of the spin. Using such a clocks-based spin measurement, we propose a variant of the Einstein–Podolsky–Rosen (EPR) gedanken experiment. To predict the outcomes of such an experiment, one needs a theory that covariantly describes the acts of spin measurement and time measurement in a relativistic quantum field theory. This is a subtle point,[32–36] which we further discuss in Appendix E. For now, we shall focus on the proposed experiment, without committing ourselves to a certain underlying theory. After presenting the gedanken experiment, we analyze it with various theories of gravity: classical and quantum. With each candidate theory, we ask whether it maintains relativistic causality in the gedanken experiment, or leads to a paradox.

The key component in our work is the intrinsic spin of all elementary particles—both general relativity and various quantum gravity theories predict that the spacetime and the intrinsic spin are coupled.[37] In particular, the spin is believed to be a source of gravity and is expected to create a minuscule aspherical curvature of spacetime, or its quantum analog; by "aspherical" we refer to a broken spherical symmetry. The next section shows how this simple symmetry breaking can be used in our EPR-like gedanken experiment. When analyzing the experiment with certain theories (including classical gravity, semiclassical quantum treatment, and also several quantum approaches), the symmetry-breaking is found to be the reason for an apparent conflict with relativistic causality, which will have to be carefully circumvented in any self-consistent theory of quantum gravity.

2. Presenting the Gedanken Experiment

The gedanken experiment is performed in three stages (**Figure 1**): At the beginning, a) An EPR pair of entangled electrons, or any spin-½ particles, $(|\uparrow_A\downarrow_B\rangle - |\downarrow_A\uparrow_B\rangle)/\sqrt{2}$ is prepared (this state is basis-invariant of course). One particle is sent to Alice and the other to Bob (Figure 1a). b) Alice decides how to orient her measurement apparatus, which for simplicity is assumed to be a pair of Stern–Gerlach magnets (Figure 1b), that is, she can orient them parallel to the $\hat{x}$ axis or she can orient them parallel to the $\hat{y}$ axis (according to her local coordinate system, see ref. [38] for a rigorous treatment of this issue). This choice splits her electron's wavefunction into a superposition of two wave-packets with spin directed along the $\hat{x}$ or $\hat{y}$ axis, with up/down orientation which can be found upon a covariant measurement. Consequently, and upon proper measurement, Bob's spin is expected to be correlated with the orientation of Alice's apparatus and with Alice's spin, that is, Bob's spin is parallel to the magnets and opposite to Alice's spin). c) Bob places extremely precise clocks at equal distances and different angles around his particle (Figure 1c), and waits long enough to be able to measure and compare minuscule differences in the hands of the various clocks that arise from time dilation.

Bob now uses the time dilations in an attempt to determine the spin axis of his electron, that is, whether his spin state is parallel to the $\hat{x}$ axis or to the $\hat{y}$ axis. This way, Bob tries to find how Alice arranged her magnets—whether she chose to orient them parallel to the $\hat{x}$ axis or parallel to the $\hat{y}$ axis.

If the spacetime curvature around his electron is correlated to its spin axis and thus breaks spherical symmetry, then Bob will be able, in principle, to use the time dilation to determine the spin axis of his electron. This contradicts relativistic causality when Alice is sufficiently far (so that Bob is outside of her light cone).

Clearly, we would not expect violations of causality to be possible. In the rest of the manuscript we shall therefore analyze how time measurement is described in different theories of gravity, trying to model the gedanken experiment without contradicting relativistic causality. We will see that the critical component in the gedanken experiment that leads to this apparent contradiction is the symmetry deviation of the spacetime curvature around the electron being correlated with the electron's spin axis. Importantly, for any consistent theoretical description of the gedanken experiment, the measurement and all the other experimental components (e.g. magnets) must be described within a relativistic quantum field theory framework.[36] Once this is done consistently, the physics underlying the gedanken experiment and all its empirical outcomes are of course independent of the kind of quantum interpretation that one uses (objective collapse, many-worlds, pilot-wave etc.).

A variant on our gedanken experiment can focus on stage (c) only, whereby Bob can attempt to "clone" the state of a particle using the spacetime measurement, thus violating the "no-cloning" theorem (thus also violating quantum unitarity).[39] Note that this single-particle description of the gedanken experiment has slightly different consequences as it does not require Alice to participate at all, focusing on only one spin state in Bob's lab. Generally, Bob can make his time measurements arbitrarily precise by accumulating time dilations over prolonged time-like intervals. The effect can also be greatly enhanced by performing the experiment simultaneously with many pairs of entangled particles.

Using the same concepts from quantum information, we propose variants of the gedanken experiment that use clocks to measure other components of spacetime[40] (e.g., elements of $g_{\alpha\beta}$ beyond g_{00}). These measurements lead to similar EPR-like tests as they are also expected to be correlated with the axis of the spin (see Section S1, Supporting Information). For another variant of our gedanken experiment that does not use spin at all, see Section S2, Supporting Information.

It is important to emphasize the differences between our clocks' EPR-type gedanken experiment and the conventional EPR gedanken experiments. So far, EPR-type gedanken experiments did not use clocks or any gravitational effects. Instead, the conventional approach is measuring the electron's spin by its magnetic interaction. In such EPR experiments, the spin state is detected through the magnetic field it creates, or by measuring its motion in response to an external magnetic field (e.g., as in a Stern–Gerlach experiment). This measurement creates a back-action effect that alters the spin state in a way that prevents finding what it was (an alternative quantum treatment of such a gedanken experiment is discussed in Section 4.3).[41] Importantly, the Stern–Gerlach experiment is not spherically symmetric—to perform this measurement, one needs to choose a specific orientation for the magnets and this choice creates a preferred axis. If the preferred axis is n, then the spin is measured with respect to the single component operator $n \cdot S$. At the end of this measurement, the state of the spin is $|n\rangle$ or $|-n\rangle$, depending on the measurement outcome of $n \cdot S$. In contrast, the clocks are placed symmetrically around the spin, therefore, there is no single preferred axis and there is no known mechanism of back-action that changes the direction of the spin.

An inherent difference in our clock-based gedanken experiment is that unlike with the magnetic field, there is no single accepted way to define neither the operators associated with measurements of spacetime nor their commutation relations. Similarly, there is no fully accepted formalism for the gravitational back-action effect from the clocks. Interestingly, several such mechanisms for describing the back-action (such as Dirac equation in curved spacetime)[42] result in a paradox if the clocks in Bob's setup are located symmetrically around the spin. More information appears in Appendix D and Section S1, Supporting Information. One key aspect of measuring the spin through the gravitational time dilation effect is the fact that (in most theories) it is independent of the spin sign and only depends on its axis (see Appendix A and note that the energy density in Einstein–Maxwell field equations (EMFE) $B^2/2\mu_0$ is sign independent). As another emphasis of the difference between a magnetic gedanken experiment and a clock-based gedanken experiment, we propose a gedanken experiment with photons instead of spins in Section S2, Supporting Information, using the same approach from quantum information. Similarly, one may conceive a version of the experiment which replaces the use of Stern–Gerlach magnets, for example, by a relativistic scattering process.[36] Either way, the resulting wavefunction splitting can be described covariantly and consistently, for example, via the proposals of refs. [36,38].

To quantify the gedanken experiment, one may try to employ a density matrix formulation that includes the clocks as part of the quantum system, so that the matrix contains the spin together with the time dilations measured by the clocks (Appendix C). Employing density matrices can be done in multiple ways, as it varies between candidate theories of quantum gravity. While some approaches can explain the gedanken experiment (e.g., Section 4.3), other approaches appear to be incapable of modeling the gedanken experiment (some mentioned as part of the outlook on other quantum approaches, e.g., the ADM formalism with the resulting Wheeler–DeWitt equation[43]). More on that below and in the Section S8, Supporting Information. In particular, further discussion of the back-action on the spin by the act of quantum measurement is in Appendix D, where we also explore a general framework that allows an entanglement of the spin with the quantized spacetime.

2.1. The Necessity of Censorship

The conceptual strength of our gedanken experiment originates from relativistic causality implying that the spacetime around Bob's electron must not indicate any deviation from spherical symmetry upon measurement of the time dilation. Any deviation of this sort would contain information about Alice's choice of basis that should be precluded on Bob's side, because it would reveal the spin axis (without its actual up/down value). This simple result points to a necessary physical mechanism that seems to be missing from the current accepted physical theory: a spin-censorship mechanism. This spin-censorship mechanism, which hinders any form of spin detection, is based on the quantum measurement of spacetime, and holds for all the elementary particles (even photons—see Section S4, Supporting Information). This notion should not be confused with the well-known cosmic censorship addressing naked-singularities. Our gedanken experiment applies far away from the spin and has nothing to do with singularities. Before we analyze different approaches to quantum gravity and discuss the existence or lack of a spin-censorship mechanism in each one, let us first show why the gedanken experiment leads to a contradiction when analyzed with classical gravity.

2.2. Testing Various Classical and Quantum Gravity Theories with the Gedanken Experiment

In the rest of this article, we neither claim to find a single unquestionable mechanism that would provide a suitable spin-spacetime censorship, nor do we favor a single theory of classical or quantum gravity. Instead, we consider models of classical and quantum gravity proposed in the literature and test in each case whether it can model our gedanken experiment or whether it leads to a paradox. The first few censorship mechanisms we consider in the discussion (part 3) are contained within classical physics. These are meant to provide some intuition toward the presented gedanken experiment, but are far from being general and problem-free.

Other classical mechanisms would modify the currently accepted theory of gravity. Specifically, such mechanisms modify the EMFE or the stress–energy tensor, which have major implications on classical observables on cosmological scales, despite the small effect of the spin-induced curvature that led to these modifications. The rest of the censorship mechanisms in the discussion (part 4) involve different ways of incorporating quantum uncertainty into general relativity, such that the act of (quantum) measurement of spacetime prevents the spin from being determined, or causes back-action on the spin. Such theories are expected to provide the required censorship mechanism without altering the (classical) theory of gravity on cosmological scales. The latter approaches are therefore more plausible, but we nevertheless present briefly the former for the sake of completeness.

Eventually, the correct censorship mechanism must be derived from the yet unknown theory that governs the interaction between quantum spacetime and matter. We show that by analyzing the requirements that the censorship mechanism must fulfill, it provides insights into how to properly describe quantum measurements of gravitational effects, which then provide new hints regarding the unified theory of quantum gravity.

3. Classical Approaches for Spin-Spacetime Censorship

3.1. Classical Approach #1—Why Classical Gravity Fails to Describe the Gedanken Experiment, Creating a Paradox with Relativistic Causality

The EMFE describe the coupling of the local spacetime curvature, expressed by Einstein's tensor $G^{\mu\nu}$ and the local stress-energy tensor $T^{\mu\nu}$, through the tensor equation $G^{\mu\nu} = \kappa T^{\mu\nu}$, where $\kappa = 8\pi G/c^4$ is Einstein's coupling constant. Applying these equations to a single electron in vacuum leads to an apparent paradox because our gedanken experiment requires $G^{\mu\nu}$ to be spherically symmetric, but the tensor $T^{\mu\nu}$ is aspherical due to the magnetic dipole moment of the electron. The axis of the magnetic dipole moment breaks the spherical symmetry of the Maxwell stress-energy tensor $T^{\mu\nu}_{EM} = \frac{-1}{\mu_0}(F^{\mu\alpha}F_{\alpha}{}^{\nu} - \frac{1}{4}g^{\mu\nu}F_{\alpha\beta}F^{\alpha\beta})$ and therefore makes the combined tensor $T^{\mu\nu}$ aspherical. The resulting spacetime is known as the Kerr–Newman solution of the EMFE (see thorough discussion in refs. [44,45]) but there are additional classical models[46] (also see Section S7, Supporting Information). The aspherical spacetime properties are apparent both in the far gravitational field (where we are not concerned with singularities) and in the near gravitational field of the electron (see **Figure 2**). As expected, the time dilation map is not spherically symmetric, which seems to enable determining the particle's spin axis by measuring minuscule time dilation differences, leading to a paradox in our gedanken experiment. To avoid the apparent conflict between the EMFE and our gedanken experiment, there should exist some specific physical mechanism (most likely a quantum one) preventing the detection of deviations from spherical symmetry.

3.2. Classical Approach #2—Modifying the Maxwell Stress-Energy Tensor

Let us begin with a classical modification of general relativity that offers a resolution for maintaining relativistic causality (in the setup of our gedanken experiment). To obtain a spherically symmetric spacetime solution, we can replace the stress-energy tensor $T^{\mu\nu}$ in the EMFE with a hypothetical tensor $S^{\mu\nu}$ that is invariant under spatial rotations in the rest frame of a single isolated electron. This replacement ensures that the Einstein tensor, now obeying $G^{\mu\nu} = \kappa S^{\mu\nu}$, is also invariant under spatial rotations and therefore ensures a spherically symmetric spacetime, which resolves the apparent paradox. A possible choice for $S^{\mu\nu}$ is the stress-energy tensor associated with dust solutions,[47] or null dust,[48,49] that is, $S^{\alpha\beta} = \sum_{v} \varepsilon(t, x; v^{\mu}) v^{\alpha} v^{\beta}$, where $\varepsilon(t, x; v^{\mu})$ is the energy density of all the particles (including charged particles, photons, and exchanged photons) passing at the point (t, x) with velocity $v^{\mu} = dx^{\mu}/dt = (1, dx/dt)$ (a quantum generalization should take into account the corresponding uncertainties). The dust stress-energy tensor includes all of the different particles' fluxes (including the photons that are exchanged between pairs of charged particles), to fulfill zero divergence $\nabla_{\alpha} S^{\alpha\beta} = 0$. For particles with nonzero rest mass, we have $\varepsilon\, v^{\alpha} v^{\beta} = \rho\, u^{\alpha} u^{\beta}$, where $\rho(t, x)$ is the proper density and u^{α} is defined as the four velocity (associated with $v^{\alpha} = dx^{\alpha}/dt$). **Figure 3** shows the resulting

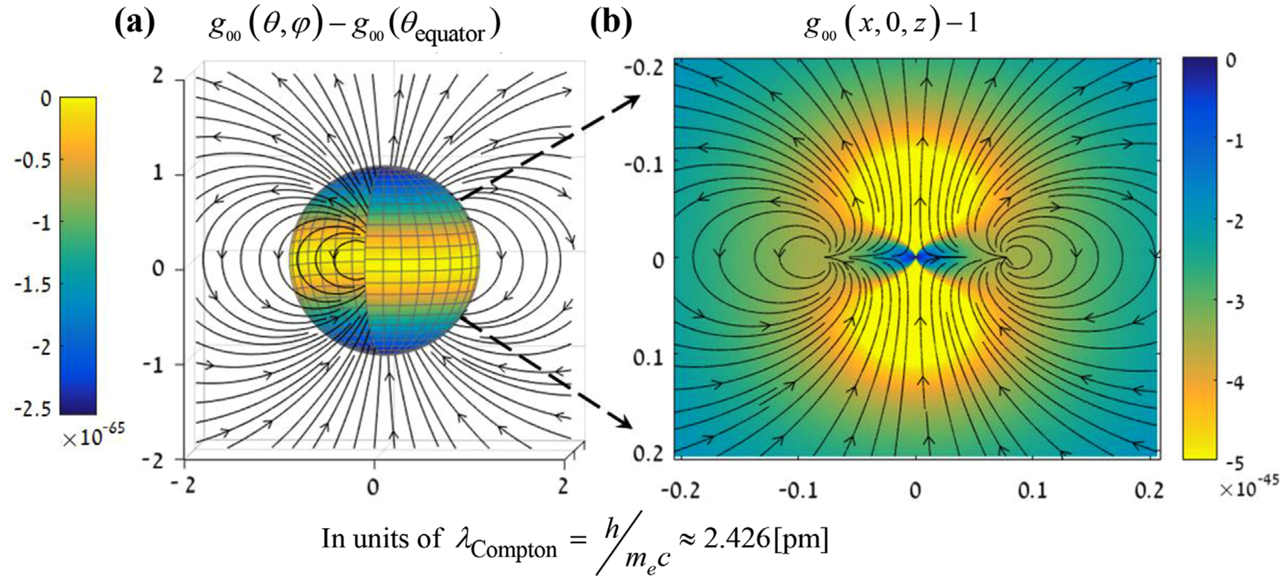

Figure 2. The extent of time dilation around an electron according to the EMFE, overlaid with its dipole magnetic field lines depicted in black: a) The aspherical part of the time dilation distribution is depicted as a color map of $g_{00}(\theta, \varphi) - g_{00}(\theta_{\text{equator}})$ on a sphere of radius 1 Compton wavelength. Clocks at the equator are ticking faster than clocks at the north/south poles, and the time dilation is maximized when the magnetic dipole field is the strongest. b) Zoomed-in plane cross section of the time dilation (depicted as a color map of $g_{00}(x, 0, z) - 1$). The time dilation is calculated according to the Kerr–Newman solution of the EMFE.

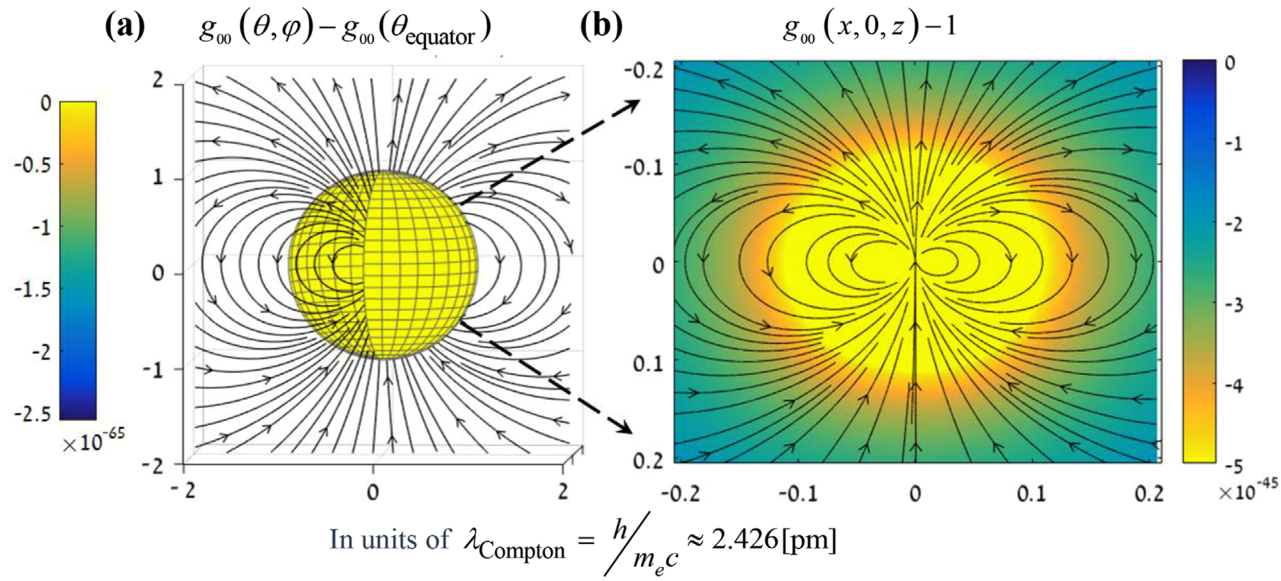

Figure 3. The measurable time dilation should be spherically symmetric to prevent (or "censor") precise assessment of the spin direction. a) A color map of $g_{00}(\theta, \varphi) - g_{00}(\theta_{\text{equator}})$ on a sphere of radius 1 Compton wavelength shows spherical symmetry, despite the magnetic dipole field lines overlaying the maps in black that are copied from Figure 1. b) Zoomed-in plane cross section of the time dilation (depicted as a color map of $g_{00}(x, 0, z) - 1$). Such a spherically symmetric spacetime may result from a modified EMFE or from a different stress-energy tensor (e.g., as in the dust stress-energy tensor approach). Here, the time dilation is calculated according to the Schwarzschild metric, while the magnetic field lines are calculated in this curved spacetime, but do not influence it.

spacetime (for a single stationary particle), which is equivalent to the spherically symmetric Schwarzschild solution. In this way the paradox is avoided. Other helpful choices of tensor $S^{\mu\nu}$, some of which contain a natural spherical symmetry, can be proposed based on different stress-energy tensors (see, e.g., the survey in ref. [50]), and further generalizations, for instance to the Kaluza–Klein theory.[51,52]

Replacing the stress-energy tensor $T^{\mu\nu}$ in EMFE with some hypothetical spherically symmetric tensor $S^{\mu\nu}$ carries significant consequences for classical physics; interestingly, it might soon be refuted in experiments: recent breakthroughs in measuring gravitational waves might bring soon the first observation of gravity waves induced by the electromagnetic stress-energy tensor (e.g., from newly born magnetars with extremely strong magnetic[53–57]).

3.3. Classical Approach #3—Adding a Torsion Tensor to EMFE

It is important to ask whether previously proposed generalized forms of the Einstein equations may already contain some forms of censorship mechanisms. Many such variants of the Einstein equations have been discussed in the literature over the past century, for example, the Einstein–Cartan theory,[58,59] which creates a coupling between the intrinsic angular momentum (classical spin) of particles and the antisymmetric part of the affine connection, known as the torsion tensor. Could the addition of torsion maintain the spherical symmetry by compensating for the particle spin? While the answer is in principle yes,[60,61] it depends on the specific torsion tensor and it is unclear whether a single torsion tensor could compensate for any arbitrary particle spin. For example, it was proven that whenever the torsion is derived from a second-rank tensor potential, static spherically symmetric solutions are not allowed,[62] thus such torsion candidates would not suffice.

One may propose other candidate censorship mechanisms within classical physics. For example, an interesting (yet at this stage very speculative) idea is the complex electromagnetic tensor[63,64] that can eliminate the aspherical parts of the Maxwell stress-energy tensor. However, it remains to be seen whether such a theory is consistent with the electromagnetic theory. See Section S5, Supporting Information, for additional candidate classical approaches such as the possible existence of the electron electric dipole moment that fail to resolve the paradox.

All the above models that avoided a violation of causality also involve subtle alterations of the accepted EMFE (or to the stress-energy tensor source term). Thus, they inevitably modify the accepted classical theory. Could there exist a quantum censorship mechanism that maintains a measurable spherical symmetry even when conventional classical gravity dictates asphericity? This question directly connects our gedanken experiment with the open questions regarding quantum measurement of spacetime in quantum gravity.

4. Quantum Approaches for Spin-Spacetime Censorship

We now turn to discuss such censorship mechanisms that have to do with quantum corrections to the classical theory. Each of them reflects on some elements and ideas pertaining to the yet-to-be-found theory of quantum gravity. More generally, we analyze whether common elements from the literature on quantum measurement of spacetime could prevent the precise inference of the spin on the quantum level, and thus facilitate the missing censorship mechanism (preferably without altering EMFE in the classical limit). This way, we note which known approaches could facilitate the missing censorship mechanism and which seem inconsistent with our gedanken experiment.

One of the difficulties in formulating a quantum gravity theory is that quantum gravitational effects only appear at length scales near the Planck scale, around 10^{-35} m, a scale far smaller, and equivalently far larger in energy, than those currently accessible by high energy particle accelerators. Therefore, physicists lack experimental data, which could distinguish between competing theories and for this reason it is important to analyze gedanken experiments to see that a specific theory does not lead to contradictions.

We explore several types of nonclassical approaches: The first approach treats spacetime as being classical, but the measurement device as being quantum, that is, it regards the possible deviation from spherical symmetry as a classical parameter which can be estimated using quantum metrology.[65] The next three approaches treat spacetime as a fully quantum object that can be described by quantum operators, that is, possible deviations from spherical symmetry are properties of the quantized spacetime that can become entangled with the spin. Here, the properties of the spacetime can be inferred using a quantum measurement of the corresponding operator, and thus they can cause back-action on the spin.

Let us add three remarks about quantum theories that we have analyzed: 1) some quantum theories can explain our gedanken experiment, for example, by quantization of the static magnetic field that determines the gravitational field of the spin in ways that maintain spherical symmetry (e.g., Section 4.3). However, 2) other quantum theories that take into account the back-action on the spin still lead to a paradox with causality (e.g., Appendix D). In particular, the perturbative approach of linearized quantum gravity, in its simple form, lacks the necessary mechanism to explain our gedanken experiment (detailed discussion in Section S8, Supporting Information). For a similar discussion on the ADM formalism, see Appendix D. In both of these cases, it would probably be possible to resolve causality paradoxes if the theories are extended to include concepts from other approaches.

4.1. Quantum Approach #1—Failure of Quantum Estimation Due to Decoherence

A process of decoherence may limit the precision of estimating the spacetime parameters, thus possibly providing a censorship mechanism. For example, a depolarizing channel introduces isotropic loss of coherence, which makes the quantum state resemble a maximally mixed state.[66] This type of noise can therefore provide a suitable censorship mechanism, if applicable during our measurement of the quantum spacetime. Another example is a dephasing channel, which naturally arises when the system is immersed in an external fluctuating field[67] and can prevent the precise spin direction inference by limiting the measurement precision. Recently, a novel decoherence process was proposed, whose rate scales exponentially with the number of particles.[68] As we show below, under certain conditions, such a strong decoherence process can mask the spin axis.

To quantify the precision limited by decoherence, we can treat the deviation from spherical symmetry, for example, an axis-dependent time-dilation dictated by the metric element g_{00}, as a small parameter λ that we wish to estimate using a quantum state ρ_N of N (possibly entangled) probing particles. Due to the measurement, the probe state undergoes some transformation $\Lambda_\lambda[\rho_N]$ yielding an estimate $\tilde{\lambda}$. For a general quantum state, there exists a bound (Cramér–Rao bound; see, e.g., ref. [69]) on the estimator's variance $V(\tilde{\lambda}) \geq 1/F_Q(\Lambda_\lambda[\rho_N])$, that uses the quantum Fisher information (QFI) F_Q. We can now maximize the QFI over all states ρ_N to find the limit on the quantum enhanced precision: $V(\tilde{\lambda}) \geq 1/F_N$, where $F_N = \max_{\rho_N} F_Q(\Lambda_\lambda[\rho_N])$. A large number N of probing particles can therefore estimate the spacetime around a single particle with increasing precision.

What can prevent the probes from finding the parameter λ with a good enough precision that enables to infer the spin? The classical Fisher information scales like N, while the QFI[65] scales like N^2, enabling, in principle, better precision. However, it was shown[69] that in the presence of decoherence, this quantum enhancement diminishes. More generally, decoherence may grow with the number of probing particles and serve to limit the precision of the estimated $\tilde{\lambda}$. Therefore, we can speculate that the probing particles exert a non-negligible effect on the measured system and on the surrounding spacetime, thereby censoring the estimation of λ or altering its value (making the scaling of the QFI much worse than N^2). A different measurement technique may involve a small number of probe particles but a prolonged probing time for increased precision in estimating the time-dilation difference. In this case, to prevent a precise estimation of λ there could be a decoherence process which grows in time as quickly as the information about λ, thus bounding the estimation precision and providing the censorship mechanism.[70]

4.2. Quantum Approach #2—Fluctuations of Spacetime

Another censorship mechanism relates the quantum uncertainty of time measurements with fluctuations of spacetime itself, upon the latter's quantization. An early version of this idea was suggested in the Quantum Foam model,[71] which was later developed to loop quantum gravity[72,73] and spin foam.[74] These models employ quantum vacuum fluctuations that may prevent a precise time measurement from indicating a deviation from spherical symmetry. The challenge here is again to find the mechanism by which fluctuations consistently overcome the signal and hide deviation from spherical symmetry even when we extend the duration of the time measurement, or repeat it many times. Then, we would be able to deduce that multiple couplings to spacetime also result in cumulative uncertainty that masks the spin value.

In light of the above, we can draw a general conclusion about the way quantum mechanics induces uncertainty into the gedanken experiment. To prevent the precise assessment of the spin axis, via repeated experiments (or one prolonged experiment), the uncertainty must grow as fast as the signal, an extraordinary behavior, as it seems to negate the Law of Large Numbers. If this behavior happens to be true in some scenarios resembling the aforementioned gedanken experiment, it may put a unique restriction on the sought-after theory of quantum gravity. It might be interesting to compare the above approach with stochastic quantum mechanics[75,76] advocating the inherent role of stochasticity in nature, conjectured to result from vacuum fluctuations.[77]

4.3. Quantum Approach #3—Expressing the Static Magnetic Field Created by the Spin by Using Spin Operators

It is well known that the axis and the direction of an unknown single spin-½ particle cannot be determined in general via measurements of the magnetic field that it creates. This limitation is normally explained via commutation relations of magnetic field components (which rely on the commutation relations of the spin components). However, it seems to be a good idea to attempt to rephrase this limitation in a different way, which can be used as a censorship approach that can help model our gedanken experiment without a paradox.

When a single spin-½ state $|+\hat{\mathbf{s}}\rangle$ is measured through its interaction $B \cdot \mu S$ with an external magnetic field $B = B\hat{\mathbf{n}}$ that is applied by a measuring apparatus, the spin can be modified by the field. The spin state $|+\hat{\mathbf{s}}\rangle$ evolves as a superposition of two non-degenerate states $|+\hat{\mathbf{n}}\rangle$ and $|-\hat{\mathbf{n}}\rangle$, with the corresponding eigenvalues of the Hamiltonian $H = B\hat{\mathbf{n}} \cdot \mu \mathbf{S}_{\hat{\mathbf{n}}} = {}^1\!/_2\, \mu B|+\hat{\mathbf{n}}\rangle\langle+\hat{\mathbf{n}}| - {}^1\!/_2\, \mu B|-\hat{\mathbf{n}}\rangle\langle-\hat{\mathbf{n}}|$, according to $|+\hat{\mathbf{s}}\rangle \to \exp(-{}^{i\hbar}\!/_2\, \mu Bt)\, |+\hat{\mathbf{n}}\rangle\langle+\hat{\mathbf{n}}|+\hat{\mathbf{s}}\rangle + \exp(+{}^{i\hbar}\!/_2\, \mu Bt)|-\hat{\mathbf{n}}\rangle\langle-\hat{\mathbf{n}}|+\hat{\mathbf{s}}\rangle$. This creates a back-action effect on the measured spin and changes its value. More generally, it is impossible to infer all the components of a single spin-½ particle, with magnetic field measurements. Recently, a new metrology technique, was proposed as a method to estimate all three components of the magnetic field,[78] but crucially, the fields were classical and quantum back-action (Appendix D) was not applicable.

In contrast with the above operators that necessitate back-action, there can be certain operators for which symmetry will prevent back-action. Below, we consider B^2 and show that while it is classically aspherical, a particular choice of quantization of the static B field (due to the spin source) will result in a spherical field. This choice will prevent the magnetic-field gedanken experiment from creating a paradox, and can also explain the spacetime-based gedanken experiment. The (classical) magnetic field created by a magnetic dipole μ is:

$$B(\mathrm{r}) = \frac{\mu_0}{4\pi}\left(\frac{3r\,(\mu \cdot r)}{r^5} - \frac{\mu}{r^3}\right) \tag{1}$$

When the magnetic dipole corresponds to a spin-½ particle, with spin operators S=(S_x, S_y, S_z) we may formally write it as,

$$\mu = \frac{g\mu_b}{\hbar}\mathrm{S} \tag{2}$$

and then the static magnetic field B is written as an operator (using Pauli matrices),

$$B(\mathrm{r}) = \frac{g\mu_0\mu_b}{4\pi\hbar}\left(\frac{3r\,(S \cdot r)}{r^5} - \frac{S}{r^3}\right) \tag{3}$$

When analyzing this expression, it is easy to see that: 1) B is not spherically symmetric in general. 2) The components of B do not commute, and hence there are nontrivial uncertainty relations between the operators B_x, B_y, and B_z, which stem from the basic uncertainty relations between Pauli matrices. Notice also that B is linear in S and hence a superposed spin leads to a similarly superposed magnetic field. The case is clearly different for measurements of an operator like B^2 (which is also what connects the field to a gravitational curvature via the energy density). For spin-½, $|B(\mathrm{r})|^2 = (\frac{g\mu_0\mu_b}{8\pi})^2(\frac{2}{r^6})I$, where we substituted $S = \frac{\hbar}{2}\sigma$ (with $\sigma = (\sigma_x, \sigma_y, \sigma_z)$ being the 2×2 Pauli matrices, and I is the identity operator). Therefore, B^2 is spherically symmetric and does not depend on the spin at all. Importantly, with further inspection, this could provide a censorship mechanism—since

$$T_{00} = \frac{1}{2}\left(\varepsilon_0 E^2 + \frac{1}{\mu_0}B^2\right) = \frac{1}{2}\left(\varepsilon_0 E^2 + \left(\frac{g\mu_b}{8\pi}\right)^2\left(\frac{2}{r^6}\right)I\right) \tag{4}$$

is spherically symmetric too and hence also $G_{00} = \frac{8\pi G}{c^4}T_{00}$. Consequently, this approach—of quantizing the static magnetic field by using the Pauli operators—may be used to explain our gedanken experiment for spin-½ particles (so far for one element of the gravitational tensor; additional elements of the stress-energy tensor and the related effects of frame-dragging require further check). Since the spherically symmetric B^2 term adds an additional $(\frac{g\mu_b}{8\pi})^2\frac{1}{r^6}$ energy term to the T_{00} stress energy tensor, this approach may lead to a new spherically symmetric electrovacuum solution associated with spin-half particles.

In contrast with the above analysis, it is currently unknown how to treat the quantitative commutation relations between operators describing the quantized gravitational field. As we do not know yet how to quantize the gravitational field in a consistent and unambiguous manner, we cannot rely on such uncertainty relations for providing a suitable spin-spacetime censorship mechanism. However, any future proposal for such a quantization method can be tested with our gedanken experiment to verify that it prevents the measurement of the spin axis.

4.4. Quantum Approach #4—Noncommutative Geometry

How else would it be possible to employ uncertainty in order to hide broken spherical symmetry? Another censorship mechanism could arise from having measurements of spacetime at different locations not commute with each other (analogous to measuring different components of the magnetic field). For example, if the time measurements by two clocks placed along the $\hat{x}$ and $\hat{y}$ axes are noncommutative, then it is impossible to measure time using these two clocks without uncertainty, thus preventing the determination of the spin axis. To quantify the amount of uncertainty, it is worth considering a noncommutative Minkowski space that can be defined in terms of spacetime coordinates x^μ, $\mu = 0, 1, 2, 3$, which satisfy commutation relations of the form $[x^\mu, x^\nu] = i\Theta^{\mu\nu}$, where $\Theta^{\mu\nu}$ is some antisymmetric tensor. This noncommutativity implies the following uncertainty relations (see, e.g., refs. [79,80]) $\Delta x^\mu \Delta x^\nu \geq \frac{1}{2}|\Theta^{\mu\nu}|$. We thus see that, similar to measurements of a quantum electromagnetic field, one has to choose a measurement basis, thus rendering the system contextual. The chosen measurement basis breaks the spherical symmetry and prevents the measurement of the spin axis (see the Section S6, Supporting Information, for further refinement arising from the case where the signaling protocol is performed with many pairs of entangled particles).

4.5. Quantum Approach #5—Quantum Decomposition into "Plane Waves"

In this section, we present a quantum approach that may suggest, in its classical limit, EMFE with a modified stress-energy tensor (see also Section 3.2). To describe the coupling of an electron to the spacetime surrounding it, we consider first a ket state that represents a stationary (zero momentum—and thus completely delocalized) electron coupled to the spacetime degrees of freedom. This stationary delocalized electron ket state is considered to be a (space-like uniform) superposition of infinitely many localized electron states. Each of these localized electron states is coupled to a quantum spacetime ket state with a specific metric corresponding to the position of the electron. We can then apply boost (affects the spacetime, as well as the electron's wavefunction) to obtain a quantum spacetime description of an electron with nonzero momentum. Finally, to construct a general quantum electron state, we use a superposition of these boosted states, creating an entangled state of the spin and the surrounding spacetime. This approach provides a candidate quantum description of an arbitrary electron state coupled to spacetimes. See Section S3, Supporting Information, for additional details. With this approach we can describe measurements of the spacetime by using a clock that measures the time dilation effects at a certain point. The clock and the electron are treated as parts of a single quantum mechanical system. Tracing out the electron degrees of freedom yields a density matrix that describes the clock. Using this density matrix, we can calculate the expectation value of the time dilation of the clock exerted by the electron's state (which is most generally described with a spinor wavefunction).

As before, measurements of time dilation via expectation values of clocks at different locations must not enable the paradoxical inference of the spin axis. Using this line of thought, we can ask ourselves—which requirement imposed on the quantum state will serve as the censorship mechanism here? Clearly, the expectation value of these measurement outcomes has to be spherically symmetric and independent of the spin axis. One possible resolution is similar to our classical approach in Section 3.2, forcing the spacetime engulfing a stationary electron to be spherically symmetric.

4.6. Outlook on Other Quantum Approaches

Many other candidate theories address the interface of gravity and quantum physics (e.g., variants of string theory) and each of them could be used to model our gedanken experiment, and then be tested by whether it creates a causality paradox. This way, our gedanken experiment serves as a tool to test the validity of such theories. For example, linearized quantum gravity[81–83] may seem as a natural candidate, but a few difficulties seem to arise when trying to apply it to our gedanken experiment (see Section S8, Supporting Information). Similarly, in view of the back-action analysis presented in Appendix D it seems unclear how loop quantum gravity[72,73] could provide a suitable spin-spacetime censorship mechanism (perhaps there could be a different mechanism which is not based on back-action).

As another example, gravitational decoherence—which in our problem means that the spacetime surrounding the electron spin "collapses" it—could affect our gedanken experiment via one of several quantitative models (e.g., refs. [84,85]). Recent advances in theories of gravitational decoherence offer intriguing thought and laboratory experiments that may resolve the conflict created by our gedanken experiment or be contradicted by it.[86–89]

The Bose–Marletto–Vedral (BMV) thought experiment[18,21] and applications thereof[90–93] could also be discussed in the context of our gedanken experiment. Their basic observation is that the mediator of quantum entanglement must be quantum in itself (provided that spontaneous collapse mechanisms like the aforementioned ones do not impede such gravitationally mediated entanglement). Our gedanken experiment here (as well as the variant in Section S2, Supporting Information, which is a bit closer to the BMV setup) is somewhat different because entanglement exists between the spins before they interact gravitationally. In the context of our work, such an approach toward quantum gravity certainly warrants a closer study. We note that Appendix D shows that entanglement between the spins and the quantized spacetime may not suffice, in themselves, for providing a spin-spacetime censorship mechanism.

A remark about theories that involve nonlinear modifications of quantum mechanics—they have general problems as even the slightest ones lead to signaling.[94–96] However, recently it was shown how to avoid this problem in certain stochastic model[97] and thus, an effective nonlinear modification could potentially be based on these approaches to study quantum spin-spacetime coupling.

Finally, it could be interesting for future work to examine our gedanken experiment in the context of the ER=EPR conjecture,[98,99] according to which our EPR pair can be thought of as being connected by a Planckian wormhole.

5. Summary

As part of the ongoing search for a consistent interface between quantum mechanics and general relativity, we have analyzed a new gedanken experiment. The gedanken experiment is concerned with the way spacetime (classical or quantum) is altered by the spin-dependent electromagnetic fields that surround entangled electrons. The gedanken experiment can be modeled by different approaches of coupling the spin with a quantized static magnetic field or with a classical/quantized spacetime (Sections 3.2, 4.2, 4.3, and 4.5). However, other approaches of classical and quantum gravity cannot model our gedanken experiment without resulting in a paradox (Sections 4.1 and 4.4, Appendix D, and Sections S5–S8, Supporting Information).

Generally, the possibility of covariant spacetime measurements that deviate from perfect spherical symmetry seems to violate relativistic causality and has led us to require a spin-spacetime censorship mechanism. Such a mechanism may lead to new restrictions on the way; spacetime is distorted due to the presence of the stress-energy tensor. We envision using the gedanken experiment as a testing ground for quantum measurement models of spacetime within classical and quantum gravity.

Appendix A: The Inability to Infer the Spin Direction from Measuring the Curvature of Spacetime

This section discusses an interesting feature of measuring the spin via the curvature of spacetime. When clocks are used to measure the time-dilation effect, various theories yield results that are the same regardless of the sign of the spin direction $|+\hat{\mathbf{s}}\rangle$ or $|-\hat{\mathbf{s}}\rangle$ (i.e., the clocks show the same result even if the spin-½ is flipped). That is, in any such theory, both of the orthogonal spin states $|+\hat{\mathbf{s}}\rangle$ and $|-\hat{\mathbf{s}}\rangle$ yield the same time dilation effect. This degeneracy can be understood, for instance, in classical gravity theories, by the time-reversal symmetry properties of the spacetime metric $g_{\alpha\beta}(x^\mu)$ (see Section S1, Supporting Information). The degeneracy of the time dilation measurement with respect to any given spanning ket basis $\{|+\hat{n}\rangle, |-\hat{n}\rangle\}$ means that this type of measurement does not act as a projection operator. However, once finding the axis without a projection, the entire spin state can be found without altering it, by placing a Stern–Gerlach magnets oriented along the spin axis (see Section S10, Supporting Information). Then, we get a contradiction with the "no-cloning" theorem. Generally, for many candidate theories, the time dilation may depend on the spin axis and not on the spin direction. It follows that in all such theories, the spin axis can be fully determined with clocks, which leads to a paradox.

This degeneracy can be understood by investigating a clock's time dilation measurement as a metrological task, being an alternative to the conventional operator-based quantum mechanical spin state measurement. Attempts to describe the measurement of the spin via the time dilation of clocks seem to circumvent the limitation imposed by the commutation relation of quantum mechanical operators. Thus, the wavefunction of a spin-½ particle remains unchanged when it is measured with clocks in this metrological manner.

To summarize this section, the measurement of the spin through time dilations differs in a fundamental way from other types of measurements such as through magnetic fields. The latter is known in quantum electrodynamics, while the former depends on the yet unknown theory of quantum gravity. Despite not having the theory, certain general conclusion can be drawn, showing the necessity of a spin-½ spacetime censorship mechanism.

Appendix B: Necessity of Spin Spacetime Censorship for Maintaining the Principle of Quantum Superposition

This section discusses the principle of quantum superposition in light of the spin-spacetime censorship. We model the gedanken experiment with a density matrix as in Appendix C, and show how even relatively general quantum mechanical considerations still require the spacetime to be independent of the spin's axis (and the spin's direction). Below, we assume that the spin and its surrounding spacetime can be described separately (as a tensor product of states), and find the resulting conditions necessary for maintaining the principle of quantum superposition. To see this, consider a spin-½ charged fermion and describe the spin states of this fermion and its corresponding time dilation by: $|\mathrm{ST}_{+x}\rangle = |+\hat{x}\rangle \otimes |\tau_{|x|}\rangle$ and $|\mathrm{ST}_{-x}\rangle = |-\hat{x}\rangle \otimes |\tau_{|x|}\rangle$, with ST standing for the combined spin-spacetime quantum state, $|\pm\hat{x}\rangle$ denoting the spin state (+½ or −½ with respect to the $\hat{x}$ axis), and $|\tau_{|x|}\rangle$ denoting the time dilation effect associated with this spin axis. On the one hand, we know that a linear superposition of these two states should just be a spin pointing at the $+\hat{z}$ axis direction—that is, $|\mathrm{ST}_{+z}\rangle = (|\mathrm{ST}_{+x}\rangle + |\mathrm{ST}_{-x}\rangle)/\sqrt{2}$, which is equal to $|\mathrm{ST}_{+z}\rangle = |+\hat{z}\rangle \otimes |\tau_{|z|}\rangle$. But on the other hand, $(|\mathrm{ST}_{+x}\rangle + |\mathrm{ST}_{-x}\rangle)/\sqrt{2} = (|+\hat{x}\rangle \otimes |\tau_{|x|}\rangle + |-\hat{x}\rangle \otimes |\tau_{|x|}\rangle)/\sqrt{2} = (|+\hat{x}\rangle + |-\hat{x}\rangle)/\sqrt{2} \otimes |\tau_{|x|}\rangle = |+\hat{z}\rangle \otimes |\tau_{|x|}\rangle$. Consequently, $|\tau_{|x|}\rangle = |\tau_{|z|}\rangle$. More generally, using all possible linear superpositions of $|\mathrm{ST}_{+x}\rangle = |+\hat{x}\rangle \otimes |\tau_{|x|}\rangle$ and $|\mathrm{ST}_{-x}\rangle = |-\hat{x}\rangle \otimes |\tau_{|x|}\rangle$, it follows that the time-dilation effect must be spherically symmetric, and thus independent of the spin. It thus seems that the spin-½ algebra poses strong requirements on the descriptions of spacetime that decouple it from the spin, rendering it spherically symmetric around elementary spin-½ particles. It seems that in order to find a self-consistent theory that allows any spacetime-spin coupling, we have to consider a mechanism by which spacetime couples to the spin direction—this is analyzed in Appendix D, together with the effect of the clock's back-action on the spin.

Appendix C: Density Matrix Considerations

This section analyzes the gedanken experiment in the language of density matrices, and discusses the underlying assumptions and their implications. Such a density-matrix-based description may be valid independently of the exact details of the unknown interaction Hamiltonian that couples the spin and spacetime degrees of freedom. Despite the generality of this description, it introduces nonlinearity at the level of the density matrix, which does not need to be the case with any model applied to our gedanken experiment.

First, let us write the density matrix in an EPR experiment: Alice uses a Stern–Gerlach device (oriented in a direction denoted by $\hat{n}$) to measure the spin and there are two possible outcomes, $|+\hat{n}\rangle$ or $|-\hat{n}\rangle$. Thus, in the ordinary EPR experiment, it is well known that, regardless of Alice's choice, the reduced density matrix describing Bob's particle is: $\rho_{\mathrm{Bob}} = {}^1\!/_2(|+\hat{n}\rangle\langle+\hat{n}| + |-\hat{n}\rangle\langle-\hat{n}|) = {}^1\!/_2 I$, that is, maximally mixed, for any choice of axis $\hat{n}$ by Alice. However, if spacetime is treated in a quantum manner and if Bob uses clocks (instead of a Stern–Gerlach device), then there are additional quantum degrees of freedom, that is, the spacetime changes due to the particle's spin.

At this point, we need to consider one detail of the interaction Hamiltonian that describes the mechanism of spin-spacetime coupling. For the rest of this section, we assume that the spacetime depends only on the spin axis, and not on its direction (±). This assumption may seem like an obvious choice at first glance, yet its consequences are significant (in Appendix D, for instance, we go beyond this assumption).

In order to measure the spin through the reading of the clocks' hands, Bob compares the delay $\tau_{|x|}$ of the clocks near the $\pm\hat{x}$ axis with the time delay $\tau_{|y|}$ of the clocks near the $\pm\hat{y}$ axis (see Section S1, Supporting Information, for discussion regarding parity symmetry of the spin's time dilation effect). To formalize the above in terms of density matrices, we assume in this section that the time dilation differences are correlated with Bob's spin axis (thus also correlated with Alice's choice of measurement) and schematically represented by the ket states $|\tau_{|x|}\rangle$ and $|\tau_{|y|}\rangle$. When Alice measures along the $\hat{x}$ axis, the reduced density matrix becomes:

$$\rho^{x}{}_{\mathrm{Bob}} = \frac{1}{2}\left(\left|\tau_{|x|}\right\rangle \otimes |+\hat{x}\rangle\langle+\hat{x}| \otimes \left\langle\tau_{|x|}\right| + \left|\tau_{|x|}\right\rangle \otimes |-\hat{x}\rangle \langle-\hat{x}| \otimes \left\langle\tau_{|x|}\right|\right) = \left|\tau_{|x|}\right\rangle\left\langle\tau_{|x|}\right| \otimes \frac{1}{2} I \tag{C.1}$$

but if she measures along the $\hat{y}$ axis, Bob's density matrix is different:

$$\rho^{y}{}_{\mathrm{Bob}} = \frac{1}{2}\left(\left|\tau_{|y|}\right\rangle \otimes |+\hat{y}\rangle\langle+\hat{y}| \otimes \left\langle\tau_{|y|}\right| + \left|\tau_{|y|}\right\rangle \otimes |-\hat{y}\rangle \langle-\hat{y}| \otimes \left\langle\tau_{|y|}\right|\right) = \left|\tau_{|y|}\right\rangle\left\langle\tau_{|y|}\right| \otimes \frac{1}{2} I \tag{C.2}$$

Importantly, the last two density matrices are no longer maximally mixed. Hence, if Bob has access to the spacetime degrees of freedom (e.g., via very precise clocks), he may use them instead of the spin degrees of freedom to decipher Alice's choice, thereby violating the no-signaling principle. Note that this violation is consistent with the literature.[86–89]

We thus find that upon measurement, the spacetime state has to satisfy $|\tau_{|x|}\rangle = |\tau_{|y|}\rangle = |\tau\rangle$. It seems hard to avoid this requirement, which also arises when testing implications of quantum superpositions (Appendix B). Such a spherical symmetry is consistent, however, with classical theories that have a spherical spacetime, such as the dust stress-energy tensor (Section 3.2), which has to modify the classical EMFE. To find a consistent theory that does not force a spherical spacetime that is decoupled from the spin, we analyze the Dirac equation in curved spacetime and the ADM formalism (see Appendix D), which however turn out to have their own limitations.

Appendix D: Quantum Back-Action Mechanisms That Fail to Provide a Suitable Spin-Spacetime Censorship Mechanism

Quantum gravitational back-action effect of the clocks on the spin-½ particle could in principle provide a suitable spin-spacetime censorship mechanism. In this section, we analyze these classical- and quantum-gravity back-action mechanisms:

- A gravitational back-action effect, from the clocks' gravitational field, acting on the spin-½ Dirac spinor's gravitational dipole moment.
- Analysis of back-action effects in quantized spacetime based on the definition of quantum operators for spacetime, as could be done with the ADM formalism, and assigning them with commutation relations that lead to the Wheeler–DeWitt equation.

Below we explain why these two back-action effects cannot provide a suitable spin-spacetime censorship mechanism.

D.1. Gravitational Back-Action Acting on the Spin-½ Gravitational Dipole Moment

According to the Dirac equation in curved spacetime, spin-½ particles are expected to have a gravitational dipole moment.[42] This theory, although well established, is currently still hypothetical, because experiments that tested it showed inconclusive results (see, e.g., ref. [96]). Furthermore, the theory is semiclassical (in the next subsection we quantize spacetime as well). Nevertheless, it is valuable to ask whether including the back-action effect of a spin-½ gravitational dipole moment could already provide the censorship mechanism, and thus model our gedanken experiment without contradicting causality. We will see below that this back-action mechanism fails to resolve the contradiction.

To explain how a back-action effect could be obtained, consider the clocks in our gedanken experiment and assume that these clocks, having spin 0, are ticking at a constant rate of ω_{Clock}. The clocks slightly bend the (single, classical) spacetime around them as they must carry energy which is at least $\hbar\omega_{\text{Clock}}$. Now consider the back-action effect of the combined clocks' gravitational field $\vec{g}_{\text{Clocks}}$ (in the rest frame of the spin-½ particle) and acting on the spin-½ gravitational dipole moment. Using the Dirac equation in curved spacetime (and neglecting high order relativistic terms in the electron's rest frame), the gravitational coupling effect on a spin-½ particle (with a spin pointing at direction $\hat{n}$), is described by an interaction term of the form[42]):

$$H_{\text{int}} = \frac{\hbar}{2c}\vec{\Sigma}\cdot\vec{g}_{\text{Clocks}}, \text{with } \Sigma_j = \begin{bmatrix} \sigma_j & 0 \\ 0 & \sigma_j \end{bmatrix} \tag{D.1}$$

with σ_1, σ_2, and σ_3 being the Pauli matrices

This interaction changes the spin direction if $\hat{n}$ and $\vec{g}_{\text{Clocks}}$ are not collinear (and $\vec{g}_{\text{Clocks}} \neq 0$). If the clock's time measurement (in our gedanken experiment) alters the spin direction through this back-action, our gedanken experiment could become analogous to the case of EPR, where Bob's measurement alters the spin and thus prevents the violation of causality. This interaction term thus seems to open a path to provide a self-consistent model to our gedanken experiment.

However, now consider the case in which the clocks are positioned symmetrically around the spin-½ particle so that the total gravitational field vanishes (i.e., $\vec{g}_{\text{Clocks}} = 0$). Such a scheme could eliminate the back-action, and thus our gedanken experiment would still result in violation of causality. In particular, we analyze the case in which Bob's spin is prepared either along the x axis or along the y axis. Bob uses symmetrically organized clocks such that the total gravitational field is zero, yet the spin axis still creates a different time dilation in the clocks depending on their positions (with respect to the spin axis—$\hat{n}$). To be concrete, consider the case, in which Bob places six clocks symmetrically around his spin located at $\vec{r}_{\text{Bob's Spin}}$, that is, the clocks are at positions $\vec{r}_{\text{Bob's Spin}} \pm L\hat{x}$, $\vec{r}_{\text{Bob's Spin}} \pm L\hat{y}$, and $\vec{r}_{\text{Bob's Spin}} \pm L\hat{z}$ (which should be understood as position expectation values since the spin and clocks are still nonclassical). At short time scales, the symmetric positioning would result in a zero gravitational field on the spin. What about longer time scales?

While initially it seems that there is no back-action, we note that due to the gravitational pull (of the spin's gravitation dipole moment) the clocks are expected to change their positions. In particular, the two clocks which are aligned along the spin axis are expected to be pulled in an asymmetric manner (one of them will get closer to the spin, while the other will get farther from it). Thus, the total gravitational field on the spin becomes nonzero.

Furthermore, the other four clocks, on the axes orthogonal to the spin, could potentially be influenced by a frame-dragging effect of the spin's gravitational dipole moment.[96] Importantly, due to symmetry considerations, these four clocks maintain a symmetric configuration around the spin and the whole configuration (six clocks and spin) remain symmetric with respect to a 90° rotation around the spin axis. Thus, the total gravitational field of the clocks, $\vec{g}_{\text{Clocks}}$ remains parallel to the spin axis. That is, if the spin is prepared in the $\pm\hat{x}$ direction, the clocks positions evolve so that the total gravitational field is $\vec{g}_{\text{Clocks}} = \pm g\hat{x}$, and if the spin is prepared in the $\pm\hat{y}$ direction, the clocks positions evolve so that the total gravitational field is $\vec{g}_{\text{Clocks}} = \pm g\hat{y}$. Most importantly, due to symmetry considerations, and even after the evolution of the clocks' wavefunctions, the spin direction $\hat{n}$ and the gravitational field $\vec{g}_{\text{Clocks}}$ remain aligned. This alignment can also be understood in another way: with angular momentum considerations. Either way, we conclude that the spin axis remains unchanged. Indeed, at all times, the spin state $|\hat{n}\rangle$ (where $\hat{n} = \pm\hat{x}$ or $\hat{n} = \pm\hat{y}$) is an eigenstate of the interaction term $\frac{\hbar}{2c}\vec{\Sigma}\cdot\vec{g}_{\text{Clocks}}$, since, $\frac{\hbar}{2c}\vec{\Sigma}\cdot\vec{g}_{\text{Clocks}}|\hat{n}\rangle = \frac{\hbar}{2c}g_{\text{Clocks}}(\vec{\Sigma}\cdot\hat{n})|\hat{n}\rangle = \frac{\hbar}{2c}g_{\text{Clocks}}|\hat{n}\rangle$.

We can summarize this subsection with the conclusion that we found a scenario in which the back-action on the spin does not alter the axis of the spin due to symmetry arguments. Therefore, it seems like Bob is able to determine the spin axis ($\pm\hat{x}$ axis, or $\pm\hat{y}$ axis) without altering this axis—which leads to a contradiction with relativistic causality. We conclude from the above discussion that the Dirac equation in curved spacetime and the theory of gravitational dipole moment do not provide a suitable spin-spacetime censorship mechanism.

Note that the same arguments above actually lead to the same conclusion in a much more general framework: It does not matter if the spin has a gravito-dipole-moment, or only a quadrupole moment, or another higher order multipole. The logic that led to the paradox works for any response to the gravitational field. Regardless of the mechanism, having a single classical spacetime that interacts with all the clocks can be used to place several clocks that cancel out the field on the spin. Thus, the above conclusion is valid beyond the Dirac equation in curved spacetime. In the next subsection, we find that similar symmetry arguments lead to similar conclusions even for fully quantized theories of gravity.

D.2. Gravitational Back-Action by Quantized Spacetime

In this subsection, we shall generalize the discussion of the previous subsection, to the case of a fully quantized spacetime. For this purpose we shall revisit the case in which Bob places six clocks symmetrically around his spin located at $\vec{\mathbf{r}}_{\text{Bob's Spin}}$, that is, the clocks are described by wave-packets located symmetrically at positions $\vec{\mathbf{r}}_{\text{Bob's Spin}} \pm L\hat{x}$, $\vec{\mathbf{r}}_{\text{Bob's Spin}} \pm L\hat{y}$, and $\vec{\mathbf{r}}_{\text{Bob's Spin}} \pm L\hat{z}$ (and of course remember that Bob's spin was prepared either along the $\hat{x}$-axis or along the $\hat{y}$-axis). Now, let us analyze the interaction of the clocks with the spin through a general framework of a quantized spacetime.

It turns out, that there is a basic symmetry argument based on angular momentum conservation, which leads to the very general conclusion: The quantization of spacetime cannot provide a back-action mechanism that can alter the spin axis provided the clocks are arranged symmetrically around it as described above. To explain the argument, consider the total angular momentum of the six spin-0 clocks with the spin-½ particle. Initially, the orbital angular momentum of the six clocks is $\vec{L}_{\text{orbital}} = 0$ and the spin's angular momentum is $\vec{S}_{\text{spin}} = {}^1\!/_2 \hbar\, \hat{n}$ for the spin-½ particle. Therefore, the total angular momentum of the system is $\vec{J}_{\text{Tot}}(t=0) = {}^1\!/_2 \hbar\, \hat{n}$. After a while, due to possible gravitational back-action effects, the spin part of the angular momentum and the orbital part of the angular momentum exchange angular momenta, such that they can become entangled. However, the total angular momentum is preserved. The state of the system is generally,

$$\left|\vec{J}_{\text{Tot}}(t)\right\rangle = \alpha_+(t)\left|\vec{S}_{\text{spin}} = \tfrac{1}{2}\hbar\,\hat{n}\right\rangle \otimes \left|\vec{L}_{\text{orbital}} = 0\right\rangle + \alpha_-(t)\left|\vec{S}_{\text{spin}} = -\tfrac{1}{2}\hbar\,\hat{n}\right\rangle \otimes \left|\vec{L}_{\text{orbital}} = \hbar\,\hat{n}\right\rangle \tag{D.2}$$

with $|\alpha_+(t)|^2 + |\alpha_-(t)|^2 = 1$.

We can no longer describe the spin state as being pure, pointing along a certain direction, since it is entangled to the quantum state of the clocks. Generally, the relative values of $\alpha_+(t), \alpha_-(t)$ should describe the spin rotation and dynamics in time (if not for its entanglement with the clocks). Nevertheless, there is no mechanism breaking the symmetry, and thus, the spin remains along the same axis. Even if the spin flips (due to gravitational back-action spin-orbital angular momenta exchange), the spin axis remains aligned with its original orientation (i.e., along the $\hat{n} = \pm\hat{x}$-axis or along the $\hat{n} = \pm\hat{y}$-axis). Furthermore, the entanglement of the spin axis with the orbital angular momentum, and through it with any additional degrees of freedom, for example, clocks positions, and quantized spacetime degrees of freedom, prevents its superposition, so it stays aligned along its original axis. We conclude that this back-action mechanism cannot provide the needed spin-spacetime censorship that would avoid the violation of causality: Bob can find out the axis of the spin by measuring the difference in time dilations between the clocks, which themselves only alter the spin direction and not its axis.

Let us now demonstrate these ideas using the ADM formalism. After applying the ADM quantization, we shall arrive to the Wheeler–DeWitt equation, which describes both matter and spacetime using one wavefunction $\Psi[h_{ij}, \varphi_{\text{Matter}}]$, where h_{ij} corresponds to the metric tensor part of the spatial foliation slices of a spacetime. That is, $g_{\mu\nu}dx^\mu dx^\nu = (-N^2 + \beta_k\beta^k)dt^2 + 2\beta_k dx^k dt + h_{ij}dx^i dx^j$ and φ_{Matter} denotes the mater state, that is, φ_{Matter} includes the electron's spinor and clocks' scalar fields.

We apply the ADM formalism to the Einstein–Hilbert action $S = \int [\frac{1}{2\kappa}R + \mathcal{L}_{\text{M}}]\sqrt{-g}\,d^4x$, where R is the Ricci scalar, $\mathcal{L}_{\text{M}}$ is a term describing particle fields, and $\kappa = 8\pi G/c^4$ is the Einstein's coupling constant. The result is a Hamiltonian of the form $H_\perp = \frac{1}{2\sqrt{h}}G_{ijkl}\pi^{ij}\pi^{kl} - \sqrt{h}\,{}^{(3)}R$, where $h = \det(h_{ij})$, $G_{ijkl} = (h_{ik}h_{jl} + h_{il}h_{jk} - h_{ij}h_{kl})$ is the Wheeler–DeWitt metric, ${}^{(3)}R$ is the induced curvature (within the spatial foliation slice), and π^{ij} are the conjugate momenta. The quantization of the ADM Hamiltonian means that the conjugate momenta are interpreted as operators satisfying $\pi^{ij}(t,x) := -i\frac{\delta}{\delta h_{ij}(t,x)}$, and having the usual quantum commutation relations with the $h_{ij}(t,x)$. Finally, the Wheeler–DeWitt equation is

$$\begin{aligned} &H_\perp \Psi\left[h_{ab}, \varphi_{\text{Matter}}\right] \\ &= \left(\frac{1}{2\sqrt{h}}G_{ijkl}\frac{\delta}{\delta h_{ij}(t,x)}\frac{\delta}{\delta h_{kl}(t,x)} - \sqrt{h}\left(\frac{1}{2\kappa}{}^{(3)}R + \mathcal{L}_{\text{M}}\right)\right) \\ &\quad \Psi\left[h_{ab}, \varphi_{\text{Matter}}\right] = 0 \end{aligned} \tag{D.3}$$

We can now use this equation to re-examine our arguments regarding the idea of back-action mechanism from gravitational dipole moment (associated with Dirac equation in curved spacetime). To do that, we consider the total four-spinor wavefunction $\varphi_{\text{Matter}} = \psi^\mu(t, x_s, x_1, x_2, x_3, \ldots, x_6)$ describing the spin-½ and the six identical spin 0 clocks around it. We note that $\psi^\mu(t, x_s, x_1, x_2, x_3, \ldots, x_6) = \psi^\mu(t, x_s, x_{p(1)}, x_{p(2)}, x_{p(3)}, \ldots, x_{p(6)})$ for every permutation $j \to p(j)$ of the clocks' coordinates, because the clocks are identical bosons. It is of course very difficult to solve this equation, but it is enough to verify the angular momentum argument on it. Due to conservation of angular momentum, if the spin is pointing at direction $+\hat{n}$ and the spin 0 clocks around it are initially stationary, the total angular momentum of the system is constant at all subsequent times and equal to $J = +{}^1\!/_2\hbar\,\hat{n}$. The next paragraph shows how conservation of angular momentum forbids the spin axis change by the back-action from the clocks.

Consider now the solution of Wheeler–DeWitt equation. In particular, consider the case that the six clocks are described by wave-packets located at positions $\pm L\hat{x}$, $\pm L\hat{y}$, and $\pm L\hat{z}$, with respect to the position of the spin (with the spin prepared at $\hat{n} = \pm\hat{x}$ or $\hat{n} = \pm\hat{y}$). Importantly, the combined state of the clocks and the spin-½ is an eigenstate of a 90° rotation around the $+\hat{n}$ axis (i.e., $R_{-90^\circ}(\hat{n})\psi^\mu(t_0, x_s, x_1, x_2, x_3, \ldots, x_6) = e^{-i\pi/4}\psi^\mu(t_0, x_s, x_1, x_2, x_3, \ldots, x_6)$, where the phase $e^{-i\pi/4}$ is due to the half integer angular momentum $J = +{}^1\!/_2\hbar\,\hat{n}$). It follows that a $\Psi[h_{ab}, \varphi_{\text{Matter}}]$ solution of Equation (D.3) is also an eigenstate of the 90° rotation around the $+\hat{n}$ axis. Furthermore, with proper preparation, the initial state of $\Psi[h_{ab}, \varphi_{\text{Matter}}]$ is an eigenstate of the unitary $R_{-90^\circ}(\hat{n})$ rotation operator. Thus, according to Equation (D.3), for any interaction of the clocks and the spin, with any induced entanglement of the spacetimes' spatial metrics h_{ab} with φ_{Matter}, the state remains an eigenstate of the unitary $R_{-90^\circ}(\hat{n})$. This simple symmetry argument proves that if the spin is located at direction $\hat{n}$, it will remain along this direction, only able to flip, but not to rotate. Therefore, Bob's measurement does not alter the spin axis.

How does Bob measure the spin axis? The time dilation measured values of the four clocks in positions perpendicular to $\hat{n}$ must all be the same. Thus, by comparing the time dilations of the clocks in $\pm\hat{x}$ and in $\pm\hat{y}$ with the clocks in $\pm\hat{z}$, Bob can find out the axis of the spin. Since this measurement is done without altering the axis, we conclude that without further alterations, the ADM formalism does not provide a suitable back-action censorship mechanism.

Appendix E: Breakdown of Relativistic Causality Due to the Act of Quantum Projective Measurement

Relativistic causality can be violated when ideal projective measurements are naively modeled in relativistic field theories using the "wave-function collapse" (or using the Lüdders rule) known from nonrelativistic quantum mechanics.[32–35] However, a consistent way to describe quantum measurements can be formulated in the framework of quantum field theory as presented in ref. [36]. Such an analysis of the proposed gedanken experiment lies beyond the scope of the current work, but several guidelines can be spelled out. The first step is to describe all of the components in our experiment (e.g., spin-½ particles, magnetic fields, and detectors) within the framework of a relativistic field theory (specifically, quantum electrodynamics).

To realize such a relativistic description, we may think about the spin-½ particles in our experiment as Dirac fields and replace the Stern–Gerlach magnets in Alice's lab (see Figure 1) with the covariant electromagnetic tensor. The governing Lagrangian is $\mathcal{L} = \bar{\psi}(i\gamma^\mu D_\mu - m)\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$, with γ^μ being the Dirac matrices, ψ the Dirac field, $D_\mu \equiv \partial_\mu + ieA_\mu$ the gauge covariant derivative, A_μ the covariant four-potential of the electromagnetic field generated by the charges, and $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ (all this can be straightforwardly generalized to curved spacetime). The detection stage can be modeled with an Unruh–DeWitt-like detector.[36,97,98] The possibility to perform such a "covariant Stern–Gerlach experiment" accords well with the analyses of refs. [38,99].

Supporting Information

Supporting Information is available from the Wiley Online Library or from the author.

Acknowledgements

The authors thank Y. Aharonov, R. Bekenstein, L. Diosi, J. D. Joannopoulos, A. Ori, S. Popescu, C. Roques-Carmes, M. Segev, M. Soljačić, and R. Wald for helpful comments and discussions. E.C. acknowledges support from the Israel Innovation Authority under projects 70002 and 73795 from the Quantum Science and Technology Program of the Israeli Council of Higher Education and from the Pazy foundation. In addition, E.C. was supported by grant No. FQXi-RFP-CPW-2006 from the Foundational Questions Institute and Fetzer Franklin Fund, a donor advised fund of Silicon Valley Community Foundation. I.K. is an Azrieli Fellow, supported by the Azrieli Foundation, and by an ERC Starter Grant (NanoEP, project number 851780).

Conflict of Interest

The authors declare no conflict of interest.

Data Availability Statement

Data sharing is not applicable to this article as no new data were created or analyzed in this study.

Keywords

entanglements, foundations of quantum mechanics, quantum information, spacetime, the EPR gedanken experiments

Received: July 29, 2021
Revised: September 7, 2021
Published online:

[1] C. Kiefer, *Ann. Phys.* **2006**, *15*, 129.
[2] M. Aspelmeyer, Č. Brukner, D. Giulini, G. Milburn, *New J. Phys.* **2017**, *19*, 050401.
[3] L. Smolin, *Three Roads to Quantum Gravity*, Basic Books, New York **2017**.
[4] L. Smolin, in *Gravity and the Quantum: Pedagogical Essays on Cosmology, Astrophysics, and Quantum Gravity* (Eds: J. S. Bagla, S. Engineer), Springer, Cham **2017**, pp. 427–450.
[5] S. W. Hawking, *Commun. Math. Phys.* **1975**, *43*, 199.
[6] A. Almheiri, D. Marolf, J. Polchinski, J. Sully, *J. High Energy Phys.* **2013**, *2013*, 62.
[7] *The Role of Gravitation in Physics: Report from the 1957 Chapel Hill Conference* (Eds: C. M. DeWitt, D. Rickles), Edition Open Access, Berlin **2011**.
[8] N. H. Lindner, A. Peres, *Phys. Rev. A* **2005**, *71*, 024101.
[9] L. J. Garay, *Int. J. Mod. Phys. A* **1995**, *10*, 145.
[10] A. Peres, D. R. Terno, *Phys. Rev. A* **2001**, *63*, 022101.
[11] R. Oeckl, *Class. Quantum Grav.* **2003**, *20*, 5371.
[12] S. Carlip, *Class. Quantum Grav.* **2008**, *25*, 154010.
[13] B. DeWitt, *The Global Approach to Quantum Field Theory*, Oxford University Press, New York **2014**.
[14] R. Penrose, *Found. Phys.* **2014**, *44*, 557.
[15] A. Mari, G. D. Palma, V. Giovannetti, *Sci. Rep.* **2016**, *6*, 22777.
[16] C. Marletto, V. Vedral, *npj Quantum Inf.* **2017**, *3*, 29.
[17] S. Hossenfelder, C. Marletto, V. Vedral, Q. gravity, *Nature* **2017**, *549*, 31.
[18] S. Bose, A. Mazumdar, G. W. Morley, H. Ulbricht, M. Toroš, M. Paternostro, A. A. Geraci, P. F. Barker, M. S. Kim, G. Milburn, *Phys. Rev. Lett.* **2017**, *119*, 240401.
[19] C. Marletto, V. Vedral, *Nature* **2017**, *547*, 156.
[20] E. C. Ruiz, F. Giacomini, Č. Brukner, *Proc. Natl. Acad. Sci. USA* **2017**, *114*, E2303.
[21] C. Marletto, V. Vedral, *Phys. Rev. Lett.* **2017**, *119*, 240402.
[22] C. Marletto, V. Vedral, *npj Quantum Inf.* **2017**, *3*, 41.
[23] M. J. W. Hall, M. Reginatto, *J. Phys. A: Math. Theor.* **2018**, *51*, 085303.
[24] A. Belenchia, R. M. Wald, F. Giacomini, E. Castro-Ruiz, Č. Brukner, M. Aspelmeyer, *arXiv:1807.07015* **2018**.
[25] T. Rothman, S. Boughn, *Found. Phys.* **2006**, *36*, 1801.
[26] F. Dyson, in *Proceedings of the Conf. in Honour of the 90th Birthday of Freeman Dyson*, World Scientific, New York **2013**, pp. 1–14.
[27] S. Popescu, D. Rohrlich, *Found. Phys.* **1994**, *24*, 379.
[28] A. Peres, D. R. Terno, *Rev. Mod. Phys.* **2004**, *76*, 93.
[29] W. K. Wootters, W. H. Zurek, *Nature* **1982**, *299*, 802.
[30] D. Dieks, *Phys. Lett. A* **1982**, *92*, 271.
[31] R. F. Werner, in *Quantum Information: An Introduction to Basic Theoretical Concepts and Experiments* (Ed: G. Alber), Springer, Heidelberg **2001**, pp. 14–57.
[32] R. D. Sorkin in *Directions in General Relativity: Volume 2: Proceedings of the 1993 International Symposium*, (Eds: C. W. Misner, B. L. Hu, D. R. Brill, M. P. Ryan, C. V. Vishveshwara, T. A. Jacobson), Maryland: Papers in Honor of Dieter Brill, Cambridge University Press **1993**, p. 293.
[33] F. Dowker, *arXiv:1111.2308* **2011**.

[34] D. M. T. Benincasa, L. Borsten, M. Buck, F. Dowker, *Classical Quantum Gravity* **2014**, *31*, 075007.
[35] B. Leron, I. Jubb, G. Kells, *arXiv:1912.06141* **2019**.
[36] C. J. Fewster, R. Verch, *Commun. Math. Phys.* **2020**, *378*, 851.
[37] Y. N. Obukhov, *Phys. Rev. Lett.* **2001**, *86*, 192.
[38] G. Flaminia, E. Castro-Ruiz, Č. Brukner, *Phys. Rev. Lett.* **2019**, *123*, 090404.
[39] V. Bužek, M. Hillery, *Phys. Rev. A* **1996**, *54*, 1844.
[40] H. Salecker, E. P. Wigner, *Phys. Rev.* **1958**, *109*, 571.
[41] More generally, spin measurements with magnetic fields cannot reveal with certainty a pre-prepared spin axis or direction, because measurements of spin operators represent quantum operators that do not commute.
[42] Y. N. Obukhov, *Phys. Rev. Lett.* **2001**, *86*, 192.
[43] B. S. DeWitt, *Phys. Rev.* **1967**, *160*, 1113.
[44] K. Rosquist, *Classical Quantum Gravity* **2006**, *23*, 3111.
[45] G. C. Debney, R. P. Kerr, A. Schild, *J. Math. Phys.* **1969**, *10*, 1842.
[46] C. A. Lopez, *Phys. Rev. D* **1984**, *30*, 313.
[47] Interestingly, this choice of $S\mu\nu$. treats the electromagnetic field as having no polarization for purposes of bending spacetime, which is an essential feature for solving the paradox. This can be intuitively understood as capturing the electromagnetic field as an incoherent superposition of waves averaged over polarization.
[48] J. Bičák, K. V. Kuchař, *Phys. Rev. D* **1997**, *56*, 4878.
[49] U. von der Gönna, D. Kramer, *Class. Quantum Grav.* **1998**, *15*, 215.
[50] C. F. Sopuerta, *Classical Quantum Gravity* **2001**, *18*, 4779.
[51] T. Kaluza, *Sitzungsber. Preuss. Akad. Wiss. Berlin (Math. Phys.)* **1921**, *966*.
[52] M. Cvetič, D. Youm, *Phys. Rev. Lett.* **1995**, *75*, 4165.
[53] B. Giacomazzo, R. Perna, *Astrophys. J., Lett.* **2013**, *771*, L26.
[54] R. Turolla, S. Zane, A. L. Watts, *Rep. Prog. Phys.* **2015**, *78*, 116901.
[55] L. Baiotti, L. Rezzolla, *Rep. Prog. Phys.* **2017**, *80*, 096901.
[56] F. Cabral, F. S. N. Lobo, *Eur. Phys. J. C* **2017**, *77*, 237.
[57] Q. Cheng, Y.-W. Yu, X.-P. Zheng, *Mon. Not. R. Astron. Soc.* **2015**, *454*, 2299.
[58] E. Cartan, *C. R. Acad. Sc. Paris* **1922**, *174*, 593.
[59] E. Cartan, *Ann. Sci. de l'Ecole Norm. Superieure* **1923**, *40*, 325.
[60] B. Kuchowicz, *Acta Phys. Pol. Ser. B* **1975**, *6*, 555.
[61] R. N. Tiwari, S. Ray, *Gen. Relativ. Gravitation* **1997**, *29*, 683.
[62] R. T. Hammond, *Class. Quantum Grav.* **1991**, *8*, L175.
[63] W. Kinnersley, *J. Math. Phys.* **1973**, *14*, 651.
[64] R. Mignani, E. Recami, *Nuovo Cimento A* **1975**, *30*, 533.
[65] V. Giovannetti, S. Lloyd, L. Maccone, *Phys. Rev. Lett.* **2006**, *96*, 010401.
[66] C. Adami, N. J. Cerf, *Phys. Rev. A* **1997**, *56*, 3470.
[67] T. Yu, J. H. Eberly, *Phys. Rev. B* **2003**, *68*, 165322.
[68] Z. Xu, L. P. García-Pintos, A. Chenu, A. del Campo, *arXiv:1810.02319* **2018**.
[69] R. Demkowicz-Dobrzański, J. Kołodyński, M. Guţă, *Nat. CommUN.* **2012**, *3*, 1063.
[70] Similar arguments can be given for repeating the experiment (possibly simultaneously) *N* times, which increases the estimation precision in our gedanken experiment (stages(a-c) above). However, this repetition cannot be applied to the version of the gedanken experiment that involves only stage (c) and seems to violate the "no cloning" theorem.
[71] J. A. Wheeler, *Phys. Rev.* **1955**, *97*, 511.
[72] C. Rovelli, L. Smolin, *Nucl. Phys. B* **1990**, *331*, 80.
[73] A. Ashtekar, *Phys. Rev. Lett.* **1986**, *57*, 2244.
[74] J. C. Baez, *Class. Quantum Grav.* **1998**, *15*, 1827.
[75] E. Nelson, *Phys. Rev.* **1966**, *150*, 1079.
[76] L. de la Peña, A. M. Cetto, A. V. Hernández, *The Emerging Quantum: The Physics Behind Quantum Mechanics*, Springer, New York **2014**.
[77] R. Tsekov, E. Heifetz, E. Cohen, *Europhys. Lett.* **2018**, *122*, 40002.
[78] T. Baumgratz, A. Datta, *Phys. Rev. Lett.* **2016**, *116*, 030801.
[79] J. W. Moffat, *Phys. Lett. B* **2000**, *491*, 345.
[80] P. Leal, A. E. Bernardini, O. Bertolami, *arXiv:1801.03767* **2018**.
[81] P. R. Anderson, C. Molina-París, E. Mottola, *Phys. Rev. D* **2003**, *67*, 024026.
[82] B. L. Hu, A. Roura, E. Verdaguer, *Phys. Rev. D* **2004**, *70*, 044002.
[83] C. J. Fewster, D. S. Hunt, *Rev. Math. Phys.* **2013**, *25*, 1330003.
[84] L. Diósi, *Phys. Lett. A* **1987**, *120*, 377.
[85] R. Penrose, *Gen. Relativ. Gravitation* **1996**, *28*, 581.
[86] I. Pikovski, M. Zych, F. Costa, Č. Brukner, *Nat. Phys.* **2015**, *11*, 668.
[87] Y. Margalit, Z. Zhou, S. Machluf, D. Rohrlich, Y. Japha, R. Folman, *Science* **2015**, *349*, 1205.
[88] A. Bassi, A. Großardt, H. Ulbricht, *Class. Quantum Grav.* **2017**, *34*, 193002.
[89] D. Kafri, J. M. Taylor, G. J. Milburn, *New J. Phys.* **2014**, *16*, 065020.
[90] F. Tamburini, I. Licata, *Phys. Lett. B* **2020**, *810*, 135792.
[91] Y. Margalit, O. Dobkowski, Z. Zhou, O. Amit, Y. Japha, S. Moukouri, D. Rohrlich, A. Mazumdar, S. Bose, C. Henkel, R. Folman, *Sci. Adv.* **2021**, *7*, eabg2879.
[92] M. Christodoulou, C. Rovelli, *Phys. Lett. B* **2019**, *792*, 64.
[93] M. Christodoulou, C. Rovelli, *Front. Phys.* **2020**, *8*, 207.
[94] a) N. Gisin, *Helv. Phys. Acta* **1989**, *62*, 363; b) N. Gisin, *Phys. Lett. A* **1990**, *143*, 1.
[95] N. Gisin, M. Rigo, *J. Phys. A: Math. Gen.* **1995**, *28*, 7375.
[96] J. Polchinski, *Phys. Rev. Lett.* **1991**, *66*, 397.
[97] a) S. Nimmrichter, K. Hornberger, *Phys. Rev. D* **2015**, *91*, 024016; b) A. Tilloy, L. Diósi, *Phys. Rev. D* **2016**, *93*, 024026.
[98] L. Susskind, *Fortschr. Phys.* **2016**, *64*, 551.
[99] M. C. Palmer, M. Takahashi, H. F. Westman, *Ann. Phys.* **2013**, *336*, 505.

