Periodic solitons in nonlocal nonlinear media

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Received August 30, 2007; accepted September 22, 2007;
posted October 4, 2007 (Doc. ID 87076); published October 29, 2007

We identify periodic solitons in nonlocal nonlinear media: multi-hump soliton solutions propagating in a fully periodic fashion. We also demonstrate recurrences and breathers whose evolution is statistically periodic and discuss why some systems support periodic solitons while others do not. © 2007 Optical Society of America

OCIS codes: 190.6135, 190.4420.

Nonlocality is an aspect of profound importance in many physical systems. In optics, nonlocal nonlinearities [1] appear when the nonlinear mechanism involves transport [2], long-range forces [3], or many-body interactions [4,5]. Interestingly, even though nonlocality generally involves some spatial averaging, even highly nonlocal nonlinearities do support solitons, which can attain a wide variety of structures, ranging from simple (bell-shape) solitons [3,6], to multihump solitons [6–8]. In all of these "nonlocal solitons" of varying structures, the underlying physical principle is that, when the nonlocal nonlinear effects are long range, the induced change in the refractive index (induced potential), Δn, depends only on the power carried by the beam P, and not on the particular structure of its intensity [1]. Hence, as proposed in [1], if the nonlocal nonlinear response is long range, then Δn induced by a given power P can support solitons of any structure whose total power is P, constructed from a superposition of the guided modes of that induced potential. These naturally include multimode solutions that propagate in a periodic fashion, arising from beating among modes, supported by Δn that is invariant in the propagation direction [1]. However, the supposition that, in highly nonlocal nonlinearities, Δn depends only on the beam power raises many intriguing questions. For example, do periodic soliton solutions truly exist, even after many periods, or do they disintegrate at some point, because in physical systems Δn does depend (even weakly) on the intensity structure? If periodic nonlocal solitons do exist, is the induced potential supporting them stationary or periodic? Are there other types of periodic nonlocal solitons whose underlying Δn is not propagation invariant? Previous studies on scalar multimode solitons in nonlocal optical systems [1,9] typically solved *linear* equations, where Δn was assumed to be stationary; hence they could not address such questions.

Here, we study nonlinear waves in a generic highly nonlocal physical system by solving for the dynamics of the wave and of the induced potential self-consistently. We identify oscillatory, high-order, scalar solitons: multihump soliton solutions that propagate in a fully periodic fashion and whose underlying induced potential is periodic as well. In addition, we demonstrate recurrences and breathers whose evolution is statistically periodic and discuss periodic solitons in systems with a limited-range nonlocality.

Consider light propagating in lead-glass displaying a thermal optical nonlinearity [6,8,10]. The heat generated by the (small) absorption diffuses, thereby increasing the temperature in a large area surrounding the illuminated region. Δn is proportional to the temperature change, thus the medium acts as a nonlocal self-focusing medium. The system is described by

$$i\frac{\partial \psi}{\partial z} + \frac{1}{2k}\nabla^2\psi + \frac{k}{n_0}\Delta n \psi = 0 \qquad (1)$$

coupled to the heat transfer equation in temporal steady state [6,8,10], which is the Poisson equation with the light intensity acting as a source

$$\nabla^2 \Delta n = -\alpha|\psi|^2. \qquad (2)$$

Here, $\psi(x,z)$ is the slowly varying amplitude of the wave, n_0 is the background refractive index, and k is the wavenumber in the material. The parameter α includes the optical absorption coefficient, the thermal conductivity, and the ratio between Δn and the temperature change [6,8]. We are now interested in solitons propagating far away from the transverse boundaries of the sample, and we therefore assume that ψ and its derivatives are all zero at the boundaries. The boundaries of the sample are kept at a constant temperature; hence $\Delta n = 0$ there.

Before demonstrating periodic solitons in our nonlocal system, it is important to understand why other conservative systems (without gain or loss) do not always support periodic solitons. The reason is power transfer from the solitons to radiation waves (unbound states of the induced potential). For example, an $N=2$ soliton is in fact two solitons that merge and split apart periodically. In the (1+1)D Kerr system, which is integrable, this dynamics does not involve any power loss to the soliton, because the induced potential is reflectionless [11]. However, in saturable systems with a local response, power is coupled from the merging beams to radiation waves every time the beams arrive at close proximity [11–13]. Hence, the solitons lose power and eventually broaden or collapse into a simple nonperiodic soliton state. Consequently, it seems logical that periodic solitons in conservative systems can exist only if the underlying nonlinear mechanism prohibits any coupling to unbound states (radiation waves), as in the integrable Kerr nonlinear Schrödinger equation, where the induced potential is reflectionless [11], or by eliminating the presence of radiation waves altogether. In what follows we show that the thermal optical nonlinearity in 1+1 dimension does not support any unbound state of the induced potential, irrespective of the optical intensity, as long as the sample boundaries are far enough from the optical beam.

The absence of radiation waves in the thermal nonlinearity is a consequence of a unique feature of this system: the depth of Δn is proportional to the width of the sample (in the transverse plane), implying that, for infinitely wide samples, the induced potential has an infinite depth. This feature is proven by formally integrating Eq. (2), which yields

$$\Delta n(x,z) = -\alpha\int_{-d}^{d}|x-x'||\psi(x',z)|^2 \mathrm{d}x' - x\frac{\alpha}{d}Px_{CM} + \alpha dP \tag{3}$$

for a sample of width $2d$, beam power $P = \int_{-d}^{d}|\psi(x,z)|^2\mathrm{d}x$, and defining the beam center (of mass) as $x_{CM} = \frac{1}{P}\int_{-d}^{d}x|\psi(x,z)|^2\mathrm{d}x$. When the beam is highly localized ($2d$ is much larger than the width of the beam), the potential depth $\Delta n_{\max} \approx \alpha dP$. Thus, in this system, the potential becomes deeper for wider samples; specifically, as $d\to\infty$ the potential depth becomes infinite, and an infinite potential well has only a discrete set of bound states, without any unbound state whatsoever. In contrast to this nonlocal system, for Kerr solitons, as well as for any solitons in saturable nonlinear media with a local response, the depth of the induced potential depends only on the optical power, but never on the sample size. Hence, systems with local nonlinear response always have unbound states. This principle is shown in Fig. 1.

It is instructive to compare the nonlocal thermal system with a related nonlocal system displaying a nonlinearity of a finite range. Such a system can be described, for example, by adding a term $-\epsilon\Delta n$ to the left-hand side of Eq. (2) [14]. The parameter ϵ is a real positive constant introducing exponential decay, thereby defining the range of nonlocality. In the limit $\epsilon \to 0$ this system yields the thermal system described by Eq. (2), whereas for very large values of ϵ the system converges to the integrable (1+1)D Kerr model [14]. We calculate Δn with this modified model, under the same conditions we did earlier when $\epsilon=0$, and plot the results in the middle column of Fig. 1. The calculation reveals that a nonzero $\epsilon\Delta n$ term, no matter how small, makes the potential noninfinite. That is, the induced potential in a nonlocal nonlinearity of a limited range is always finite; hence the system inherently always has unbound states.

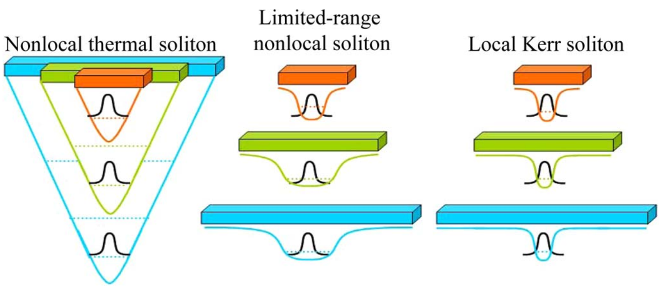

Fig. 1. (Color online) Comparison between the induced potentials in a local Kerr medium (right), a limited-range nonlocal nonlinearity (middle), and an infinite-range nonlocal (thermal) nonlinearity (left). Shown is the potential induced by a soliton of a given width in three samples of different sizes (rectangles). For each sample, the potential well is drawn with the lowest energy level (dotted lines), and the soliton (black curves) self-trapped within it. In local media, the induced potential is not affected by the sample size (as long as the sample is much wider than the soliton). The same occurs for finite range nonlocal nonlinearity, as long as the sample is much wider that the nonlocality range. However, in the nonlocal thermal medium, the potential depth is proportional to the sample width, thus becoming infinitely deep as the sample width goes to infinity.

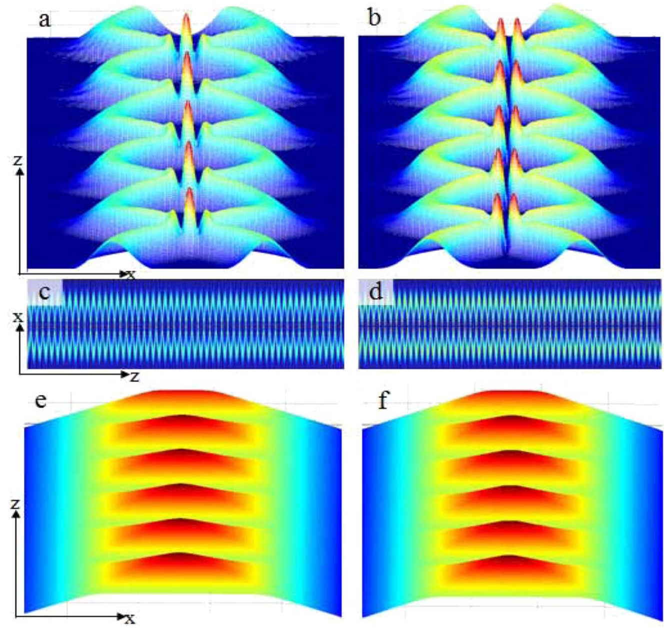

Fig. 2. (Color online) Periodic scalar solitons of the (1+1)D thermal system. a, b, First five periods, for an input relative phase of 0 and π, respectively. c, d, Same periodic solitons propagating over a very large distance. e, f, The induced potential $\Delta n(x,z)$ for the solitons of a and b. The propagation dynamics is stable and fully periodic at all distances.

The absence of unbound states (radiation modes) in the nonlocal thermal system has profound implications. Specifically, during soliton collisions, radiation waves provide a pathway for the escape of power from the interaction region. In the (1+1)D realization of our system, radiation waves do not exist; hence power cannot escape from solitons during their interaction. Thus, solitons undergoing periodic collisions do not lose power, and indeed, under proper parameters, stable periodic soliton states exist.

Following this logic, we generate (numerically) periodic solitons in our system from two identical parallel-launched stationary solitons. Examples shown in Fig. 2 depict scalar solitons whose multihump structure oscillates periodically. The initial separation between the solitons is ~6 times a soliton width, and the relative phase between the solitons is 0 (Figs. 2a and 2c) or π (Figs. 2b and 2d). We find this periodic state by solving the coupled set of Eqs. (1) and (2) without further assumptions. Its existence is the result of the energy conservation and nonexist-

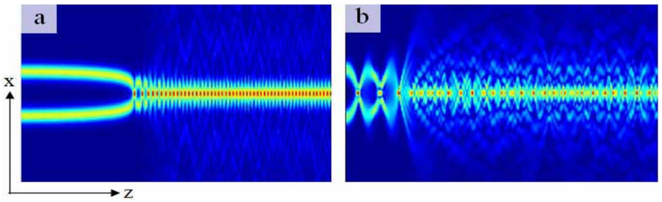

Fig. 3. (Color online) Two parallel in-phase solitons launched, a, in a saturable nonlinearity with a local response, and, b, in the limited-range nonlocal nonlinearity described by adding the term $-\epsilon\Delta n$ to Eq. (2). b, In both cases, the solitons attract each other and merge (fuse) into a lower mode while emitting radiation.

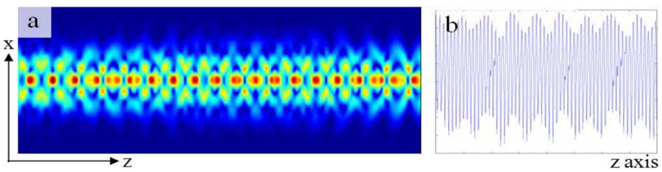

Fig. 4. (Color online) High-order scalar soliton displaying stable quasi-periodic propagation dynamics. The soliton is composed of the first and third modes propagating in a quasi-periodic manner. a, Intensity structure of the scalar high-order soliton. b, The autocorrelation function, calculated for a longer propagation distance, exhibits a statistically periodic structure.

ence of radiation waves, while its stability is due to the phase-independent attraction between widely separated solitons. Figures 2e and 2f reveal that Δn oscillates periodically, in unison with the optical intensity (Figs. 2a and 2b). In other words, Δn supporting these nonlocal periodic solitons varies periodically during propagation, as opposed to Δn in [1], which was assumed to be propagation invariant.

We now examine periodic solitons in local saturable nonlinearities and in limited-range nonlocal nonlinearities. As an example for saturable nonlinearities, consider $\Delta n \propto |\psi|^2/(1+|\psi|^2)$, representing, e.g., homogeneously broadened two-level systems, the photorefractive screening nonlinearity, and the photovoltaic nonlinearity. We launch the periodic state by bringing two in-phase solitons to close proximity. As expected [13,15], these solitons merge (fuse), as shown in Fig. 3a. When the solitons arrive at close proximity, power escapes into radiation waves; hence the propagation dynamics is never periodic; rather, the two-soliton state decays into a single, nonperiodic, soliton. Next, we seek periodic solitons for the nonlocal nonlinearity represented by Eq. (2) with a nonzero $-\epsilon\Delta n$ term on the left-hand side. As we did for Fig. 2, we bring two in-phase solitons to close proximity. Alternatively, we bring two solitons with any relative phase to within a distance much larger than the width of a soliton but shorter than the range of nonlocality. In both cases, the solitons attract each other, either (in the first case) because of coherent attraction or (in the second case) because of the phase-independent attraction between them [6–8]. The outcome (Fig. 3b) is always the same: the periodic state couples power to radiation waves and collapses into a nonperiodic state. Evidently, including the $-\epsilon\Delta n$ term in Eq. (2) precludes the possibility of periodic solitons in limited-range nonlocal nonlinear media (except perhaps when both ϵ and α are very large and the system is close to the Kerr limit).

Finally, in addition to solitons with perfectly periodic dynamics, we find multihump solitons with quasi-periodic propagation. These are constructed from several bound states of the induced potential. An example is shown in Fig. 4a, where we launch modes 1 and 3 of Δn, weighted with equal power, and with a relative phase of $\pi/2$. Such quasi-periodic solutions conserve their total power and maintain a self-similar structure, although they never repeat their actual wave function. Apparently, such quasi-periodic solitons can be constructed from two (or more) periodic states whose period ratio is irrational. The autocorrelation function calculated for the presented state is statistically periodic (Fig. 4b): the spectral contents of the oscillation are the same for all z, but their relative weights vary stochastically.

In conclusion, we demonstrated periodic solitons for the thermal optical nonlinearity, which has a very large range of nonlocality. The periodic solitons arise because this system lacks radiation waves, i.e., all possible states of the induced potential are localized, forming a discrete set. Consequently, soliton interactions in this system are never accompanied by power leakage to radiation modes. This unique feature gives rise to stable periodic and quasi-periodic self-localized states. Following similar logic, we conjecture that our system might also exhibit other features of integrable systems, such as conservation laws for interacting solitons.

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