Accelerating diffraction-free beams in photonic lattices

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We study nondiffracting accelerating paraxial optical beams in periodic potentials, in both the linear and the nonlinear domains. In particular, we show that only a unique class of z-dependent lattices can support a true accelerating diffractionless beam. Accelerating lattice solitons, autofocusing beams and accelerating bullets in optical lattices are systematically examined.

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In the last few decades, the physics of diffractionless optical beams [1–4] has attracted considerable attention because of their intriguing physical properties and their potential applications. More recently, a new class of diffraction-free beams was introduced: Airy beams [5,6]. Such beams are unique solutions of the paraxial wave equation in one-dimensional homogeneous media. Not only are they diffraction-free, but more interestingly they self-accelerate (spatially bend) even in free space. Unlike other solutions that propagate on a straight line (e.g., Bessel beams), they follow a parabolic trajectory while maintaining their shape. Naturally, one could ask if such beams can also exist in inhomogeneous media and, in particular in periodic potentials [7–10]. Indeed, it has been shown theoretically [7] and experimentally demonstrated [8] that the Wannier–Stark (WS) beams can accelerate in propagation-invariant optical lattices (i.e., when the potential does not depend on the evolution coordinate). Additionally, an important development occurred last year, with the discovery of accelerating nondiffracting solutions of the full Maxwell's equations [11], which were subsequently demonstrated experimentally [12], with beams following circular [12,13] and elliptic [14] trajectories. Recently, nonparaxial accelerating beams were also found inside photonic crystals [15]. However, it is important to note that even though the families of such beams in periodic structures [7–10,15] can experience self-acceleration, they are not truly diffraction-free; rather, the shapes of these beams self-reconstruct while moving through the periodic potential in a bending trajectory. An interesting question thus arises: are self-accelerating and true diffraction-free optical beams possible in general inhomogeneous media? In other words, what is the analog of Airy beams when a periodic potential is involved? Can one extend these ideas to the nonlinear domain, to obtain accelerating lattice solitons? We investigate these issues in this Letter.

To do so, we consider the nonlinear normalized Schrödinger equation, that governs the paraxial nonlinear wave dynamics in the presence of an optical potential $V(x,z)$, which is

$$iU_z + U_{xx} + V(x,z)U + \gamma|U|^2U = 0, \tag{1}$$

where $U(x,z)$ is the electric field envelope, z is the propagation distance, x is the transverse coordinate axis, and γ is the Kerr nonlinear coefficient. The subscripts in Eq. (1) denote the partial derivatives with respect to z and x. To address our question, we follow an analytical method for deriving any self-accelerating and diffractionless solution of the paraxial equation of diffraction in both linear and nonlinear regimes. We apply the moving frame approach [16] where in the accelerating-moving coordinate system $(\bar{x},z)$, the new coordinate $\bar{x}$ is defined as $\bar{x} \equiv x - F(z)$, with $F(z)$ being the unknown trajectory of the diffractionless beam. In particular, Eq. (1) can be written in the new coordinate system as: $iU_z - iF'(z)U_{\bar{x}} + U_{\bar{x}\bar{x}} + V(\bar{x},z)U + \gamma|U|^2U = 0$. The $F'(z)$ stands for the derivative of the trajectory with respect to z. By using the ansatz $U(\bar{x},z) = \phi(\bar{x})e^{i\theta(\bar{x},z)}$ and assuming real optical potential $V(\bar{x},z)$ we obtain a set of two coupled nonlinear partial differential equations:

$$\phi_{\bar{x}\bar{x}} + [F'(z)\theta_{\bar{x}} - \theta_z - \theta_{\bar{x}}^2 + V(\bar{x},z)]\phi + \gamma\phi^3 = 0, \tag{2a}$$

$$[2\theta_{\bar{x}} - F'(z)]\phi_{\bar{x}} + \theta_{\bar{x}\bar{x}}\phi = 0. \tag{2b}$$

Close inspection of Eqs. (2) reveals that, for our solution to be diffractionless and accelerating, two conditions must be satisfied:

$$F'(z)\theta_{\bar{x}} - \theta_z - \theta_{\bar{x}}^2 + V(\bar{x},z) = \text{function of } \bar{x} \text{ only}, \tag{3a}$$

$$\theta_{\bar{x}} = \frac{1}{2}F'(z) = \text{function of } z \text{ only}. \quad (3b)$$

Equation (3a) assures that the beam is diffraction-free since the potential term in Eq. (2a) will be a function of $\bar{x}$ and therefore the beam is stationary in the moving frame of coordinates. Equation (3b) can be rigorously derived by integrating Eq. (2b). In particular, by multiplying Eq. (2b) with ϕ we get $(\partial/\partial\bar{x})[\theta_{\bar{x}}\phi^2 - (1/2)F'(z)\phi^2] = 0$. Integration of this equation together with the physically reasonable boundary condition $\lim_{\bar{x}\to\pm\infty}\phi(\bar{x}) = 0$ leads us directly to Eq. (3b). Next we proceed to solve Eq. (2a) based on the conditions of Eqs. (3a) and (3b). Integration of Eq. (3b) yields $\theta = (1/2)F'(z)\bar{x} + c_1(z)$. The last formula can be then used to obtain the derivative of θ with respect to z: $\theta_z = (1/2)F''(z)\bar{x} + c_1'(z)$. Substituting the above relations to Eq. (3a) we arrive at

$$\frac{F'^2(z)}{4} - \frac{F''(z)}{2}\bar{x} - c_1'(z) + V(\bar{x}, z) = \text{function of } \bar{x}. \quad (4)$$

It immediately follows that, for Eq. (4) to be satisfied (given the fact that V is periodic in x), the optical potential $V(\bar{x}, z)$ in the accelerating frame system should be a function of $\bar{x}$ only, meaning that $V(\bar{x}, z) = V(\bar{x})$. This means that the potential in the lab coordinate system should be z-dependent $V = V(x, z)$. Therefore, diffraction-free and accelerating beams in propagation-invariant optical lattices are, rigorously speaking, impossible. That is, for beams to exhibit shape-invariant propagation in periodic structures, the potential should also depend on z. Therefore, for z-dependent optical potentials, Eq. (4) can be rewritten as: $\{0.25F'^2(z) - c_1'(z)\} - 0.5F''(z)\bar{x} =$ function of $\bar{x}$. In order for this equation to be satisfied we must have $4c_1'(z) = F'^2(z)$ and $F''(z) = c_2$. Integration of the latter equation leads us to $F(z) = c_3 z^2$. Based on this relation for the trajectory $F(z)$, we can calculate $c_1(z)$ and then find the unknown function $\theta(\bar{x}, z)$. After few steps the result we obtain is $\theta(\bar{x}, z) = c_3\bar{x}z + (c_3^2/3)z^3$. By changing the notation of c_3 to $a \equiv c_3$, we can write the trajectory $F(z)$ and the phase $\theta(\bar{x}, z)$ as $F(z) = az^2$, $\theta(\bar{x}, z) = a\bar{x}z + (a^2z^3/3)$. By substituting the expressions for $F(z)$ and $\theta(\bar{x}, z)$ into Eq. (2a), we can find the general solution of Eq. (1), which is

$$V(x, z) = V_0(\bar{x}) - \lambda, \quad (5a)$$

$$U(x, z) = \phi(x, z)\exp[i(axz - 2a^2z^3/3)], \quad (5b)$$

$$\phi_{\bar{x}\bar{x}} + [V_0(\bar{x}) - a\bar{x}]\phi + \gamma\phi^3 = \lambda\phi, \quad (5c)$$

where $V_0(\bar{x})$ represents a given optical potential in the moving frame, and λ is a real constant. More specifically, for the z-dependent potential $V(x, z) = V_0(\bar{x}) - \lambda$,, the diffraction-free and accelerating beam $U(x, z)$ that satisfies Eq. (1) is given by Eq. (5b), where $\phi(x, z)$ is an eigenmode of Eq. (5c) in the moving frame with corresponding eigenvalue λ. Evidently, it is clear from Eqs. (5), that the only acceptable trajectory for a self-accelerating and diffraction-free beam in one-dimensional periodic potential is the parabolic one $F(z) = az^2$ (as in the case of the Airy beam in free space). More specifically, for a parabolically bending lattice $V_0(x - az^2)$, the modes of the WS Hamiltonian [17] $\partial_{\bar{x}\bar{x}} + V_0(\bar{x}) - a\bar{x}$ (corresponding to eigenvalue λ) of the shape-invariant lattice $V_0(\bar{x})$ in the moving frame are the only accelerating and diffraction-free beams of the lab-frame Hamiltonian $\partial_{xx} + V_0(x - az^2)$. The phase dependence is that of the Airy beam in free space [5,6]. The same connection between the WS lattice solitons in the moving frame and the accelerating solitons in the lab frame holds in the nonlinear domain.

At this point we have to note that for the special case of free space $V = 0$, Eq. (5c) reduces to the well-known Airy equation $\phi_{\bar{x}\bar{x}} - \bar{x}\phi = 0$, which leads to the Airy beams [5,6]. Likewise, more general diffractionless and accelerating beams with nonparabolic trajectories of the type $F(z) = cz^2 + z^3/3$ or $F(z) = cz^2 - 2\sin z$ can be constructed by assuming more general forms of the optical potentials of the type $V(\bar{x}, z) = V_0(\bar{x}) + A(z)\bar{x} + B(z)$; these lead to more complicated potentials in the lab frame, which we are not going to consider here.

For demonstration purposes and without any loss of generality, we consider one spatial dimension even though our results are valid for any type of periodic potential in both spatial dimensions. We consider the potential $V_0(\bar{x}) = A\cos^2(\bar{x})$ in the moving frame of reference, which corresponds to $V(x, z) = A\cos^2(x - az^2) - \lambda$ in the lab frame. The potential is z-dependent [18] and periodic for every value of the propagation distance z (insets of Fig. 1). Such solution exists only for a parabolic $F(z) = az^2$ trajectory. In other words, these parabolically bent lattices are the only ones supporting diffraction-free

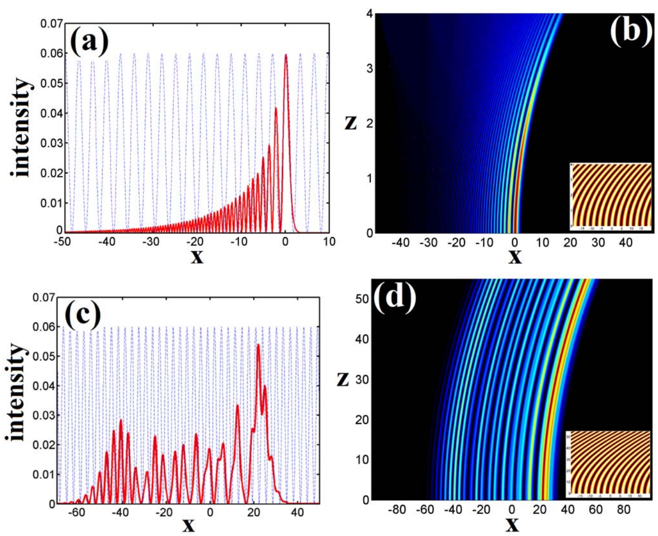

Fig. 1. (a) Truncated WS beam (solid lines) of the lattice in the moving frame for $a = 1$ at $z = 0$ (the dotted lines represent the waveguide channels). (b) Linear propagation dynamics of the beam of (a) in a z-dependent linear potential. (c), (d) Same quantities as in (a), (b) but for a lattice with $a = 0.01$. Both insets illustrate the z-dependent lattice in the lab frame (the vertical axis: propagation distance z, horizontal axis: transverse coordinate x).

and accelerating optical beams in one spatial dimension. Any other bent lattice would lead to radiation losses.

Figures 1 illustrate the field profiles at the input $z = 0$ [Figs. 1(a) and 1(c)] and the diffraction dynamics [Figs. 1(b) and 1(d)] in the corresponding z-dependent lattice, for the two cases of $A = 1$, $\alpha = 1$ [Figs. 1(a) and 1(b)] and $A = 1$, $\alpha = 0.01$ [Figs. 1(c) and 1(d)], respectively. Evidently, in both cases the beams accelerate along parabolic trajectories and they do not experience any diffraction. Therefore, these beams are diffractionless and accelerating. The bending of the lattice itself also follows a parabolic trajectory, as shown in the insets in Figs. 1(b) and 1(d). Even though the lattice is bent (z-dependent), there are no radiation losses upon propagation. The linear coefficient α controls the spatial distribution of the WS modes. In the first case, the WS modes resemble Airy beams since the linear refractive index tilt $\alpha = 1$ is high with respect to the refractive index difference $A = 1$ (related to the coupling between neighboring channels), and the input WS profile is exponentially truncated. In the second regime, when the linear tilt is much smaller than the parameter A, the WS modes are spatially confined and still asymmetric in their intensity profile since they are composed of higher bands Floquet–Bloch modes. We have to note at his point that the center of mass of our beams does follow the parabolic trajectory.

The above results can be extended to the nonlinear regime, for $\gamma = 1$ (self-focusing). In this case, the diffraction-free and accelerating solutions are localized because of the nonlinearity and are given by Eqs. (5), where λ represents the soliton eigenvalue (nonlinear phase shift). In Fig. 2(a) we can see a highly confined accelerating lattice soliton for $A = 1$, $\alpha = 0.001$ and in Fig. 2(b) the dynamics of the same soliton in a lattice that follows the $x = (0.035z)^{1.5}$ trajectory that bends less than the parabolic one. We can see that the soliton is breathing upon propagation and experiences radiation losses, since only in the unique class of parabolically bended potentials diffraction-free and accelerating solitons exist. Thus, we have identified a unique correspondence between WS lattice solitons of the moving frame and accelerating self-trapped beams in a parabolically bending periodic potential. The nonlinear accelerating beams recently introduced in [16] are completely different physical entities from the accelerating lattice solitons we studied above. Their existence bifurcation curves, stability properties as well as their field profiles are not the same since the underlying linear problem is different for the free-space and the lattice geometry.

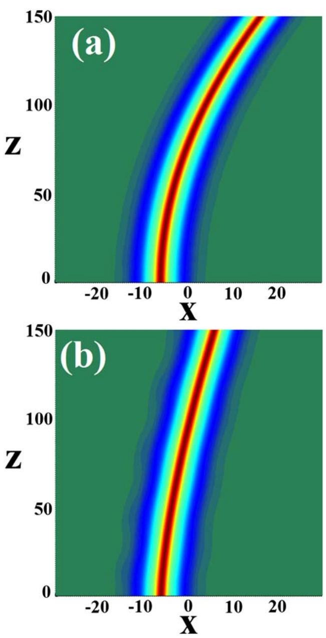

Fig. 2. (a) Highly confined accelerating lattice soliton for $A = 1$, $\alpha = 0.001$ in a parabolically bended lattice and (b) dynamics of the same soliton in a lattice that follows the different trajectory $x = (0.035z)^{1.5}$. In both cases the nonlinearity is self-focusing.

Thus far, we have examined the existence and the evolution of diffractionless and accelerating beams in parabolically bent periodic potentials. A physically relevant question is what happens when the lattice is invariant in z and is not bent. This question has been previously examined within the context of coupled-mode theory, and it would be of interest to extend this study beyond this tight binding regime. For low refractive index contrast, higher band effects become more profound and the accelerating and diffracting WS modes are characterized by asymmetric intensity lobes, as illustrated in Fig. 3(a). As such, the z-dependence of the lattice is crucial for the diffraction-free character of these WS beams.

Next we demonstrate, that in stationary (propagation-invariant) lattices, we can use such accelerating WS beams to construct autofocusing beams [19,20] and spatiotemporal Airy–Bessel discrete bullets. In particular, a judicious superposition of two WS modes, can give us an autofocusing beam in a stationary lattice. The appearance of an abrupt focus inside the lattice is illustrated in Fig. (3(b)). The optical intensity at the focus plane is confined to a few waveguide channels. In contrast to autofocusing beams constructed by caustics engineering [9,10], autofocusing in the present scenario arises from a carefully designed superposition of the modes of two WS-Hamitonians $\partial_{\bar{x}\bar{x}} + \cos^2(\bar{x}) \pm 0.01\bar{x}$.

Finally, we consider propagation of accelerating optical pulses in uniform periodic potentials. Within the coupled-mode theory approximations, the diffraction dynamics is governed by the normalized equation $i(\partial u_n/\partial z) + (\partial^2 u_n/\partial t^2) + u_{n+1} + u_{n-1} = 0$, where the second term accounts for dispersion, t is the time, and u_n the field amplitude at the nth channel [7]. In this regime one can derive the following analytic expression,

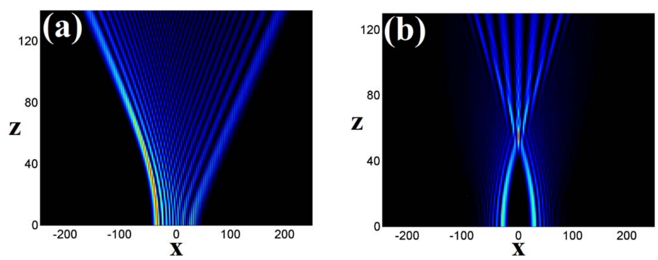

Fig. 3. Diffraction dynamics of (a) a single WS mode and (b) an autofocusing beam (superposition of two WS modes) in a propagation-invariant lattice with $A = 1$, $\alpha = -0.01$. In both cases the optical intensities of the propagating beams are depicted for various values of the propagation distance.

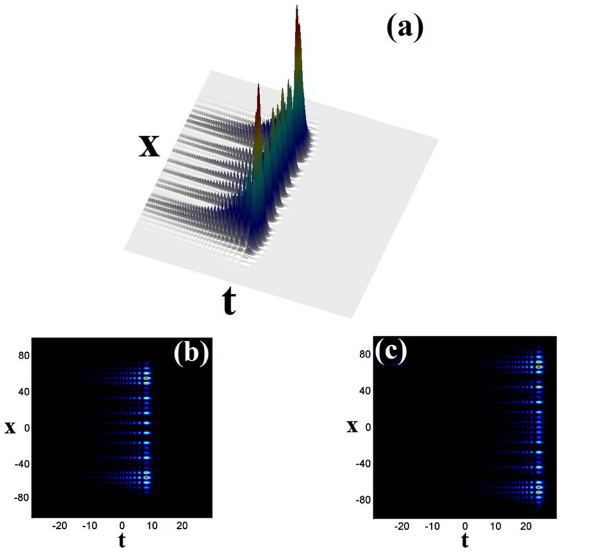

Fig. 4. (a) Intensity profile of the accelerating optical bullet at $z = 0$, and top view profiles for propagation distances (b) $z = 3$, and (c) $z = 5$.

by applying the separation of variables method, in terms of Bessel and Airy functions: $u_n = J_{-n}(2\sqrt{z^2 + b^{-2}})e^{-in\tan^{-1}(bz)}Ai(t - z^2)e^{itz - i\frac{2}{3}z^3}$, where b is the parameter defining the spatial extent of the beam [5,6]. As we can see, such solution is the product of a WS accelerating beam [7] and a temporal Airy beam [5,6]. These spatiotemporal beams are accelerating and dispersionless in time and accelerating in space. The optical field therefore forms a discrete bullet confined in time and space. It is interesting to note that the spatial acceleration of this bullet follows a hyperbolic trajectory while its temporal profile experiences dispersion-free parabolic acceleration. Intensity profiles of such a solution for different propagation distances are depicted in Fig. 4.

In conclusion, we investigated the properties of accelerating and/or diffraction-free beams in optical lattices. Our analysis indicates that diffractionless and accelerating beams can only exist in z-dependent bending periodic potentials with a spatial profile of a Wannier–Stark mode and the phase of an Airy beam. Our results hold in the linear as well as in the nonlinear domains where these beams are accelerating lattice solitons. We also examined autofocusing effects arising from superposition of WS modes and demonstrated the possibility of discrete accelerating bullets in uniform photonic lattices.

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